

Symbolic Model Checking Property Specification Language*

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Agenda



Motivations

- Srief Overview of Symbolic Model Checking of LTL
- From PSL to APSL
- Tableau based Model Checking of APSL
- BDD based Encoding of APSL Tableaux
- Tool and Experiments
- Conclusion

Model Checking



- Model checking is an automatic technique to verify if a system satisfies its design specification.
- Why model checking?
 - Complexity of modern hardware/software.
 - Impractical for manually verification and proving.
- How to run model checking?
 - Model, abstraction of systems, described as (finite) transition systems, Kripke structures
 - Specification, property to be verified
 - Checking!

Specification Language



- Specifications are written with various temporal logics.
- Specification Language) has become an industrial standard (IEEE 1850).
 - Evolved from some industrial used language, such as VHDL & Verilog.
 - Has full expressiveness to describe all the omega-regular properties.

Properties such as "*p* holds at every even moment" cannot be expressed by any LTL formula.

Verification of PSL



Approaches

- Bustan, Fisman and Havlicek developed an automata-based approach for model checking PSL.
- Tuerk, Schneider and Gordon presented PSL model checking using HOL and SMV.
- Pnueli and Zaks developed a model checking approach, based on testers.
- Tools
 - The tool RuleBase of IBM
 - Zeroln of Mentor

Motivation



How to efficiently model check PSL?

- Symbolic Model Checking PSL
 - A BDD-based symbolic approach for PSL model checking
 - Achieve the goal without doing too much adaptation to the existing popular verification tools

Basic Idea

Property Specification Language.



Basic Idea

Clarke et al presented an adapted LTL model checking framework, which converts the LTL MC problem to that of CTL







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Models



A model is a state-labeled transition system (or, Kripke structure)

$$M = \langle S, \rho, I, L, \Omega \rangle$$

where:

- S is a (finite) set of states.
- $\rho \subseteq S \times S$, is the transition relation.
- $I \subseteq S$, is a set of initial states.
- L: S $\rightarrow 2^{AP}$, is the labeling function.
- $\Omega \subseteq 2^{S}$, is a set of fairness constraints.

Computations



The linear perspective of a model



Computations



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The branching perspective of a model



Linear Temporal Logic



& LTL



LTL Semantics



LTL Semantics

-
$$\pi$$
, $i \models p$ iff $p \in \pi(i)$.
- π , $i \models \neg \varphi$ iff π , $i \models \varphi$ not holds.
- π , $i \models \varphi_1 \land \varphi_2$ iff π , $i \models \varphi_1$ and π , $i \models \varphi_2$.
- π , $i \models X\varphi$ iff π , $i+1 \models \varphi$.
- π , $i \models \varphi_1 \cup \varphi_2$ iff there is some $j \ge i$, s.t. π , $j \models \varphi_2$ and for each $i \le k < j$ s.t. π , $k \models \varphi_1$.

Model checking problem of LTL

 $-M \models \varphi$ iff all fair derived paths of *M* satisfy φ .

Computation Tree Logic

CTL

$$\varphi ::= p$$

$$| \neg \varphi$$

$$| \varphi \land \varphi$$

$$| AX\varphi$$

$$| A(\varphi \cup \varphi)$$

$$| E(\varphi \cup \varphi)$$

Semantics of CTL formulae is defined on computation trees.

The CTL model checking problem $M \models \varphi$ is to verify that if all computation trees unwounded from *M* satisfy φ .

PDI

CTL Model Checking



Framework of CTL model Checking (explicit)



CTL Symbolic Model Checking PDL®

Idea of symbolic model checking

- Using n bit variables to represent 2^n states.



We can use 2 bit variables v_0 and v_1 to encode the states.

CTL Symbolic Model Checking pot



Each subset of the state set corresponds to a Boolean function over v_0 and v_1 . For example, the state set { s_1 , s_2 }, in which p is evaluated to *true*, can be represented as $\neg v_1$.

CTL Symbolic Model Checking PDL®



Each transition can be characterized as a Boolean over v_0 , v_1 , v'_0 and v'_1 . e.g., the transition corresponding to the red edge can be written as $v_0 \land \neg v_1 \land v'_0 \land v'_1$. Use the disjunction of such formulae as the encoding of the transition relation.

CTL Symbolic Model Checking PDL

- Initial states, fairness constraints and the labeling (for each proposition) can be encoded into a Boolean function.
- What we need to storage is a set of Boolean formulae, instead of explicit states and transitions.
- With these formulae, we can compute, in a bottom-up manner, the Sat set of each subformula.
- This computing process is manipulated based on BDDs (Binary Decision Diagrams), which can be implemented in an efficient way.

Symbolic Model Checking of LTL pot



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	r ::= b	Boolean expression			
0	<i>r</i> ; r	concatenation			
	<i>r</i> : <i>r</i>	fusion			
C	r r	choice			
	r && r	and			
	r*	Kleen closure			
2	<i>r</i> @c	clock sampling			

Examples

Concatenation: r_1 ; r_2





Clock Sampling: r@c



Semantics of FL



Semantics of FL formulae

L(r)

Ψ

 $r \mathsf{T} \varphi$

- Formulae of b, $\neg \varphi$, $\varphi_1 \wedge \varphi_2$ are defined as usual
- Formulae of $X\varphi$ and $\varphi_1 U \varphi_2$ are defined as same as in LTL
- π , $i \models (\varphi \text{ abort } b)$ iff either π , $i \models \varphi$ or there is some $j \ge i$, and some π' , s.t. $\pi[i,j]; \pi' \models \varphi$ and $\pi, j+1 \models b$

-
$$\pi$$
, $i \models r \top \varphi$ iff there is some $j \ge i$, s.t. $\pi[i,j] \in L(r)$, and $\pi, j \models i$

Symbolic model checking PSL? PDL*

- Symbolic model checking PSL?
 - The major effort of PSL model checking must put on that for FL formulae.
 - Explore the idea of LTL symbolic model checking.

Elementary Formulas

- * For each formula φ , we can inductively construct the set of elementary formulae of φ , denoted $El(\varphi)$ as follows:
 - $El(b) = \{ p \in AP \mid p \text{ occurs in } b \};$
 - $El(\neg \psi) = El(\psi);$
 - $El(\psi_1 \wedge \psi_2) = El(\psi_1) \cup El(\psi_2);$

$$- El(\mathbf{X}\psi) = \{\mathbf{X}\psi\} \cup El(\psi);$$

 $- El(\psi_1 \cup \psi_2) = \{ \mathsf{X}(\psi_1 \cup \psi_2) \} \cup El(\psi_1) \cup El(\psi_2) \}$

* An element of $El(\varphi)$ is either an atomic proposition or a formula of the form $X\psi$

PDL

Sat functions



- * For each subformula ψ of φ , define the function *Sat*, which maps ψ to a set of subsets of $El(\varphi)$.
 - Sat $(p) = \{W \subseteq El(\varphi) \mid p \in W\};$ Sat $(\neg \psi) = 2^{El(\varphi)} \setminus Sat(\psi);$

 - Sat $(\psi_1 \wedge \psi_2) = \operatorname{Sat}(\psi_1) \cap \operatorname{Sat}(\psi_2);$
 - $\operatorname{Sat} (X\psi) = \{W \subseteq El(\varphi) \mid X\psi \in W\};$

- Sat
$$(\psi_1 \cup \psi_2) =$$

Sat $(\psi_2) \cup (Sat(\psi_1) \cap Sat(X(\psi_1 \cup \psi_2))).$

Tableau



* The tableau of φ , denoted T_{φ} is the transition system $\langle S_{\varphi}, \rho_{\varphi}, I_{\varphi}, L_{\varphi}, \Omega_{\varphi} \rangle$, where:

- S_{φ} consists of subsets of $El(\varphi)$.
- $(W,W') \in \rho_{\varphi}$ iff for each $X\psi \in El(\varphi)$, $W \in Sat(X\psi)$ if and only if $W' \in Sat(\psi)$.

$$-I_{\varphi} = Sat(\varphi).$$

 $-L_{\varphi}(W) = W \cap AP.$

- For each subformula $\psi_1 \cup \psi_2$ of φ , there is a fairness constraint $Sat(\neg(\psi_1 \cup \psi_2)) \cup Sat(\psi_2)$ in Ω_{φ} .

For each
$$\pi \in (2^{AP})^{\omega}$$
, $\pi \models \varphi$ iff $\pi \in L(T_{\omega})$

Tableau of FL?



For FL formula, the difficulty is that the transition structure is not explicit. So, when defining the *Sat* function for formula of r T b, it is hard to write an explicit formula.

Replace SEREs with NFAs



✤ A variant of PSL, namely APSL.







$$\varphi ::= b$$

$$|\neg \varphi | \varphi \land \varphi$$

$$|X\varphi | \varphi \cup \varphi$$

$$|A \text{ abort! } b \text{ strongly abort}$$

$$|A \top \varphi \text{ automaton trigger}$$

Semantics of AFL



The semantics

- π , $i \models A$ abort! b iff there is some $w \in PreL(A)$ and some $j \ge i$, s.t. $\pi[i,j] = w$ and $\pi, j+1 \models b$.
- $-\pi$, *i* ⊨ A⊤*φ* iff there is some *j* ≥*i*, s.t. π[*i*,*j*] ∈*L*(A) and π, *j* ⊨ *φ*.
- AFL and FL have precisely the same expressiveness.
- Study symbolic model checking problem for AFL.

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Elementary Formulae of AFL

Given an NFA $A = \langle \Sigma, Q, \delta, Q_0, F \rangle$, we denote by A^q the NFA $\langle \Sigma, Q, \delta, \{q\}, F \rangle$.

Elementary formula set of AFL formula

- For the formulae of b, $\neg \psi$, $\psi_1 \land \psi_2$, $X\psi$, $\psi_1 \cup \psi_2$, their elementary formula sets are defined same as before.
- $El(A \text{ abort! } b) = El(b) \cup \{X(A^q \text{ abort! } b) \mid q \text{ is a state of } A\}.$

 $- El(A \mathsf{T} \psi) = El(\psi) \cup \{\mathsf{X}(\mathsf{A}^q \mathsf{T} \psi) \mid q \text{ is a state of } A\}.$

PDL

Sat function of AFL formulae



Sat(A^q abort! b) = Sat(b) $\cup \{W \subseteq El(\varphi) \mid \exists r \in \delta (q, W \cap AP), \text{ s.t. } X(A^r \text{ abort! } b) \in W\}$ $\pi, i \models A^q \text{ abort! } b$ iff either $\pi, i \models b$, or $\pi, i \models X(A^r \text{ abort! } b)$ for some $r \in \delta(q, \pi(i))$

Sat function of AFL formulae

Sat function of T

Sat($A^q \mathsf{T} \psi$) $\{W \in Sat(\psi) \mid \delta(q, W \cap AP) \cap F \neq \emptyset\} \cup$ $\{W \subseteq El(\varphi) \mid \exists r \in \delta (q, W \cap AP), \text{ s.t. } X(A^r \top \psi) \in W\}$ $\pi, i \models A^q \mathsf{T} \psi$ iff either π , $i \models \psi$ and $\delta(q, \pi(i)) \cap F \neq \emptyset$ or π , $i \models \mathbf{X}(A' \top \psi)$ for some $r \in \delta(q, \pi(i))$

Tableaux for AFL formulae



* Given an AFL formula φ , suppose that (A_1, ψ_1) , ..., (A_m, ψ_m) are all its trigger pairs, and A_i 's state set is Q_i . The tableau of φ is a special transition system

$$\mathbf{T}_{\varphi} = \langle \mathbf{S}_{\varphi}, \rho_{\varphi}, \mathbf{I}_{\varphi}, \mathbf{L}_{\varphi}, \Omega_{\varphi} \rangle$$

- S_{φ} consists of tuples $\langle W, (P_1, \dots, P_m) \rangle$, where $W \subseteq El(\varphi)$ and $P_i \subseteq Q_i$.

$$- (\langle \mathsf{W}, (\mathsf{P}_1, \dots, \mathsf{P}_m) \rangle, \langle \mathsf{W}', (\mathsf{P}'_1, \dots, \mathsf{P}'_m) \rangle) \in \rho_{\varphi} \text{ iff }$$

- For each $X\psi \in El(\varphi)$, $W \in Sat(X\psi)$ iff $W' \in Sat(\psi)$;
- For each $1 \le j \le m$, $((W, P_j), (W', P'_j)) \in \rho_{(Aj, \psi)}$.

$$-I_{\varphi} = \{ \langle W, (P_1, \dots, P_m) \rangle \mid W \in Sat(\varphi) \}.$$

$$-L_{\varphi}(\langle W, (P_1, \ldots, P_m) \rangle) = W \cap AP.$$

Trigger Pair / Transition



Given an AFL formula φ , we say that (A, ψ) is a trigger pair in φ , if there is some q, such that $A^q T \psi$ is a subformula of φ .

Assuming $A = \langle 2^{AP}, Q, \delta, Q_0, F \rangle$, the trigger pair (A, ψ) of φ derives a trigger transition relation $\rho_{(A,\psi)} \subseteq (2^{El(\varphi)} \times 2^{Q})^2$ such that $(W_1, Q_1, W_2, Q_2) \in \rho_{(A,\psi)}$ iff \Rightarrow If $Q_1 = \emptyset$, then $Q_2 = \{q \mid W_2 \in Sat(A^q T \psi)\}$. \Rightarrow If $Q_1 \neq \emptyset$, then for each $q \in Q_1$, - either $W_1 \in Sat(\psi)$ and $\delta(q, W_1 \cap AP) \cap F \neq \emptyset$ - or there is some $q' \in Q_2$, s.t. $q' \in \delta(q, W_1 \cap AP)$

Trigger Transition Relation POL





Fairness Constraints



- * Ω_{φ} consists of three parts:
 - For each subformula $\psi_1 \cup \psi_2$, create a fairness constraint $\{\langle W, (P_1, ..., P_m) \rangle |$

 $W \in Sat(\psi_2) \text{ or } W \notin Sat(\psi_1 \cup \psi_2) \}.$

For each subformula A^q abort! b, create a fairness constraint

 $\{\langle \mathsf{W}, (\mathsf{P}_1, \dots, \mathsf{P}_m) \rangle \mid$

W \in Sat(b) or W $\notin \bigcup_{r \in Q}$ Sat(A^r abort! b)}.

- For each trigger pair (A_i, ψ_i) , create a fairness constraint

 $\{\langle \mathsf{W}, (\mathsf{P}_1, \ldots, \mathsf{P}_m) \rangle \mid \mathsf{P}_i = \emptyset \}.$

AFL Model Checking



Theorem (Language Property of AFL tableaux):

For each
$$\pi \in (2^{AP})^{\omega}$$
, $\pi \models \varphi$ iff $\pi \in L(T_{\varphi})$

AFL model checking problem is converted to that of CTL.

 $M \models \varphi$ iff $(M || T_{\varphi})$ does not satisfy EG *true*

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- With the BDD based technique, product of two transition systems can be naturally implemented.
- BDD encoding of the original model can be acquired from the users' input.
- How to obtain the encoding of an AFL formula's tableau.



Given an AFL formula φ with trigger pairs (A₁,ψ₁), ...,(A_m,ψ_m) and the state set of A_i is Q_i.
Recall that a state in the tableau is a tuple ⟨W,(P₁, ... P_m)⟩, where W ⊆ El(φ) and P_j⊆ Q_j. Then:
For each ψ∈ El(φ), create a Boolean variable u_ψ.
For each 1≤i≤m and each q ∈ Q_i, create a Boolean variable u_ψ.

variable $v_{(i,q)}$.

* For each subformula ψ of φ , we may build a Boolean formula f_{ψ} , which characterizes $Sat(\psi)$.



- The symbolic encoding of transition relation is the conjunction of the following issues:
 - For each $X\psi \in El(\varphi)$, add a conjuct $u_{X\psi} \Leftrightarrow f_{\psi}$.
 - For each $1 \le i \le m$, employ the following two conjuncts $(\bigwedge_{q \in Q_i} \neg v_{(i,q)}) \Rightarrow \bigwedge_{q \in Q_i} (v'_{(i,q)} \Leftrightarrow f'_{A_i^q \top \psi_i})$

If $Q_1 = \emptyset$, then $Q_2 = \{q \mid W_2 \in Sat(A^q T \psi)\}$



* For each subformula $\psi_1 \cup \psi_2$, we add a fairness constraint encoding

$$f_{\psi^1 \cup \psi^2} \lor f_{\psi^2}.$$

* For each subformula A^q abort! *b* with $A = \langle \Sigma, Q, \delta, Q_0, F \rangle$, we add the fairness constraint encoding

 $f_b \vee \bigwedge_{q' \in Q} \neg f_{A^{q'} \text{abort}!b}$

• For each trigger pair (A_i, ψ_i) , we create the fairness constraint encoding

 $\wedge_{q \in Qi} \neg v_{(i,q)}.$

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Tool Support



From NuSMV to ENuSMV

NuSMV is a symbolic model checking tool by CMU/ ict-IRST, and it supports both CTL and LTL model checking.

- The extended version of SMV is adapted from NuSMV Ver 2.4.3.
- ENuSMV Ver 1.0 support ETL model checking, and Ver 1.1 support APSL.
- Available at <u>http:// enusmv.sourceforge.net</u>

Automata Constructs



Defining Automata constructs in ENuSMV



CONNECTIVE $A(a_1,a_2)$ STATES: > $q_1, q_2 <$ TRANSITIONS(q_1) case $a_1: \{q_1,q_2\};$ $a_2: q_2;$ esac;

Philosopher Dining



- Feasibility: each philosopher can possibly have a meal.
- Liveness: it is possible for a philosopher to eat infinitely many times.
- Machine Specification
 - CPU: Intel Core Duo2 (2.66GHz)
 - Memory-size: 2G

Philosopher Dining



Feasibility						
	# states		#time (sec.)			
#philosophers	CTL	AFL	CTL	LTL	AFL	
6	1135	2270	0.020	0.046	0.046	
7	3545	7090	0.037	0.122	0.124	
8	11395	22790	0.070	0.310	0.311	

Philosopher Dining



Liveness						
#philosophor	# states		#time (sec.)			
#philosopher	CTL	AFL	CTL	LTL	AFL	
6	1135	4412	0.021	0.040	0.046	
7	3545	14180	0.0367	0.073	0.074	
8	11395	45580	0.070	0.150	0.149	

Verification of Non-star-free Properties



- Consider the model of "accumulative carry circuit", which is a synchronous circuit composed a set of modulo 2 counters.
- These counters are connected in a series manner the first counter's input is set to 1, and the (*i*+1)-th counter's input is connected to the *i*-th counter's output.

Accumulative Carry Circuit

PDL

MOUDLE counter(carry_in) VAR

value: boolean; pre_value: boolean; ASSIGN

init(value):=0;

init(pre_value) :=0;

next(pre_value) :=value;

next(value) := (value +carry_in) mod 2;

DEFINE

carry_out := (pre_value & carray_in);

Accumulative Carry Circuit



bit_0: counter(1);

bit_i+1: counter(bit_i.output);

bit_n: counter(bit_n-1.output);

Accumulative Carry Circuit

Consider the following properties:

- Periodicity: bit_0 carries out at each odd moment.
- Monitoring: once the waveform matches the pattern true; (bit_0.carry_out)*, then the signal bit_1.carry_out would raise in the next two steps.

None of the above properties is star-free.

Cannot be verified with the previous NuSMV.

Accumulative Carry Circuit pol



	Periodicity		Monitoring		
#bits	# states	#time	#bits	# states	#time
7	66560	0.011	7	8320	0.009
8	132096	0.012	8	16512	0.011
9	263168	0.019	9	32896	0.018
10	525312	0.024	10	65664	0.021

Other Experiments



- To test the scalability, we have done DME (Distributed Mutual Exclusive) circuit experiment. Results show that ENuSMV can handle models with more than 5×10²¹ reachable states.
- The experiment on security policy of SE-Linux shows that ENuSMV can handle a model larger than 120,000 lines.
- Notably, for the same property, AFL model checking sometimes has a better performance than that of LTL, especially when the connective nesting depth is large.

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Conclusion



- We have presented a variant of PSL, namely APSL, which has precisely the same expressive power as omega-regular expression.
- We extended the BDD-based symbolic model checking algorithm to that of APSL.
- A symbolic model checker supporting APSL is designed and implemented based on NuSMV.



Thank you. Questions and Comments?