Assume-Guarantee Reasoning – Overview

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Motivation

- Assume-guarantee reasoning can alleviate the state explosion problem in model checking.
- There are two major issues:
  - How to devise proper assumptions?
  - How to develop new proof rules?
- Interestingly, these two issues may be related to program analysis.
- Hopefully, we can combine static analysis and model checking in new perspectives.

Outline

1. Assume-Guarantee Reasoning
2. Algorithmic Learning Theory
3. Our Related Works
   - Learning Regular ω-Languages
   - Learning Minimal Separating Finite Automata
   - Intuitionistic Interpretation
4. Conclusions

Automata-Theoretic Framework I

- Let M denote a system (a program, protocol, or even circuit).
- Let P denote a property (mutual exclusion, starvation freedom).
- $M \models P$ (informally) means "the system M satisfies the property P."
- In early days, various formalizations of the $\models$ relation were proposed.
- Automata-theoretic framework was introduced in the 80's by Moshe Vardi.
  - It is widely used in model checking community now.

Automata-Theoretic Framework II

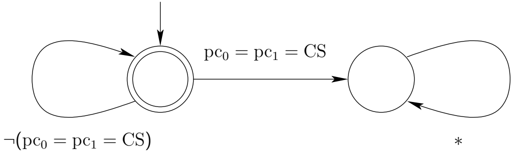
- Let M be a finite automaton specifying a system.
  - System behaviors are strings accepted by M.
- Let P be a finite automaton specifying a property.
  - Intended behaviors are strings accepted by P.
- Let A be a finite automaton. Define

$$\mathrm{L(A)} = \{\alpha : \alpha \text{ is accepted by A}\}.$$

- Then $\mathrm{M} \models \mathrm{P}$ can be formalized as $\mathrm{L(M)} \subseteq \mathrm{L(P)}$.

Automata-Theoretic Framework III

- Many model checking problems can be reduced to formal language problems.
- For instance, suppose M_P is Peterson's algorithm.
- To check whether M_P satisfies mutual exclusion, consider the following automaton P_{ME}:



- Finite automata can be constructed from a logic formula.
  - For instance, $G(pc_0 \neq CS \vee pc_1 \neq CS)$.

Automata-Theoretic Framework IV

- We can also consider a composition $M_0 \| M_1$ of systems M_0 and M_1.
- For instance, consider $M_0 \| M_1$ to have all behaviors shared by M_0 and M_1.
  - Thus, $L(M_0 \| M_1) = L(M_0) \cap L(M_1)$.
- Concise specifications of systems and properties are possible.
  - For instance, M_{P_0} and M_{P_1} specify the processes in Peterson's algorithm.
  - We can check if $M_{P_0} \| M_{P_1} \models P_{ME}$.
- However, the number of states grows exponentially in the number of processes.
  - This is called the state explosion problem.

Compositional Reasoning

- Compositional reasoning aims to solve verification problems by divide and conquer.
- Consider the following proof rule:

$$\frac{M_0 \models P}{M_0 \| M_1 \models P}$$

  - Informally, if we can prove M_0 satisfies P, then $M_0 \| M_1$ satisfies P.
- It is easy to see that the rule is sound. But it is useless in practice.
  - Each process in Peterson's algorithm does not satisfy mutual exclusion alone.
- Contexts must be considered in compositional reasoning.

Assume-Guarantee Reasoning

- In assume-guarantee reasoning, contextual assumptions are provided by users. Consider

$$\frac{M_0 \| A \models P \qquad M_1 \models A}{M_0 \| M_1 \models P}$$

  - Informally, if we can find an assumption A such that (1) $M_0 \| A$ satisfies P; and (2) M_1 satisfies A, then $M_0 \| M_1$ satisfies P.
- Intuitively, assumptions are abstractions of contexts.
- With proper abstractions, problems are divided into simpler subproblems.

Issues in Assume-Guarantee Reasoning

However, there are two issues in assume-guarantee reasoning:

1. How to come up with "proper assumption?"
   - Finding assumptions requires insights to system designs.
     - Try to find an assumption to prove Peterson's algorithm.
   - Is there a way to generate assumptions automatically?
     - Yes! I will cover this exciting idea here.
2. How to find new proof rules?
   - Some proof rules are rather perplexing. Consider

$$\frac{M_0 \| P_1 \models P_0 \qquad P_0 \| M_1 \models P_1}{M_0 \| M_1 \models P_0 \| P_1}\ \mathcal{C}$$

   - Is there a way to verify these proof rules automatically?
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Algorithmic Learning Theory

- Let U be the universe.
- Let $L \subseteq U$ be an unknown set.
- With the help of a "teacher," a learning algorithm aims to find a finite representation R such that R represents L.
- For instance, consider U to be the set of finite strings and L a regular language. Suppose a teacher gives (α_i, c_i) such that

$$\begin{array}{ll} \alpha_i \in L & \text{if } c_i = \text{true} \\ \alpha_i \notin L & \text{if } c_i = \text{false}. \end{array}$$

- A learning algorithm aims to computer a finite automaton representing L by observing (α_i, c_i) for $i = 0, \ldots, N$.

Advices are Important

- Learning algorithms depends on types of teachers heavily.
  - In the previous slide, a learning algorithm cannot be correct if the teacher always gives the same advice (say, (ϵ, true)).
- It is necessary to formulate problems carefully in algorithm learning theory.
  - Improper formulations can lead to infeasible solutions or even make the problem unsolvable.

The L* Algorithm

- In 80's, Angluin introduced the L^* algorithm for learning an unknown regular language L.
- The L^* algorithm can make two types of queries:
  1. Membership: is the string α in L?
     - ★ The teacher answers "yes" or "no."
  2. Conjecture: is the finite automaton C accepting L?
     - ★ If $L(C) = L$, the teacher answers "yes."
     - ★ If $L(C) \neq L$, the teacher gives a counter example in $(L(C) \setminus L) \cup (L \setminus L(C))$.
- Given such a teach, L^* generates the minimal finite automaton M such that $L(M) = L$ with a polynomial number of queries (in the size of M).

Why does Machine Learning Matter?

- Notice that a teacher in the L^* algorithm does not need any insight to the unknown language.
- Consider the language $L = 1^*(1^*01^*01^*)^*1^*$.
- By examining the regular expression, we realize that L contains all finite strings with an even number of 0's.
  - To see this, certain insights are needed.
- However, checking $\alpha \overset{?}{\in} L$ or $L(C) \overset{?}{=} L$ can be mechanized.
  - By translating the regular expression to a finite automaton.
- If we can simulate a teacher in assume-guarantee reasoning, the L^* can learn proper assumptions for us!

Assume-Guarantee Reasoning through Learning I

- Consider again the proof rule:

$$\frac{M_0 \| A \models P \qquad M_1 \models A}{M_0 \| M_1 \models P}$$

- For membership query $\alpha \overset{?}{\in} L(A)$, we check if $\alpha \in L(P) \cup \overline{L(M_0)}$.
- For conjecture query $L(C) \overset{?}{=} L(A)$, we check if $M_0 \| C \models P$ and $M_1 \models C$.
  - If yes, C is a proper assumption;
  - If no, check whether the counter example is in $L(M_0 \| M_1) \setminus L(P)$.
    - If yes, P is falsified by the counter example;
    - If no, returns the counter example to L^* to refine C.
- Eventually, either
  1. the assumption A such that $L(A) = L(P) \cup \overline{L(M_0)}$ is generated; or
  2. a counter example is found.

Assume-Guarantee Reasoning through Learning II

- In principle, the unknown language is the weakest assumption $L(P) \cup \overline{L(M_0)}$.
  - Membership queries are resolved accordingly.
- However, we may find a proper assumption before reaching the weakest assumption.
  - Any assumption A such that $L(M_1) \subseteq L(A) \subseteq L(P) \cup \overline{L(M_0)}$ will do.
- The idea is not to generate assumptions intrinsically but externally.
  - It is impossible to "guess" contextual assumptions.
  - It is nevertheless possible to "refine" them incrementally.

Our Related Works

- Since the introduction of machine learning to assumption generation in 2003, several papers have been published in this area.
- We are interested in the following problems:
  1. Extending the framework to regular ω-languages;
  2. Finding better assumptions;
  3. Deriving more proof rules automatically.

Learning Regular ω-Languages I

- For liveness properties ("good things will happen"), it is more natural to use infinite strings (called ω-strings).
- One can discuss ω-strings accepted by a finite automaton.
  - The problem is to define accepting conditions since there is no final state in an infinite run.
- A finite automaton thus accepts set of ω-strings (called an ω-language).
- In 60's, Büchi showed that the class of ω-languages accepted by finite automata coincides with the class of regular ω-languages.
- In model checking community, regular ω-languages are used very often.

Learning Regular ω-Languages II

- In order to extend assume-guarantee reasoning through learning to liveness properties, we need a learning algorithm for regular ω-languages.
- However, L^* depends on the Myhill-Nerode theorem.
  - It is also used to prove the existence of minimal deterministic finite automaton for regular languages.
- But there is no similar theorem for finite automata over ω-strings.
  - Most intriguingly, deterministic and non-deterministic finite automata may have different expressive power over ω-strings.

Learning Regular ω-Languages III

- Our idea is to represent regular ω-languages by regular languages.
- For any ω-language L, define

$$\mathrm{L}_\sharp = \{\mathrm{u}\sharp\mathrm{v} : \mathrm{uv}^\omega \in \mathrm{L}\}.$$

- If L is a regular ω-language, then $\mathrm{L}_\sharp$ is a regular language.
- For any unknown regular ω-language L, we use L^* to learn $\mathrm{L}_\sharp$.
- When the finite automaton $\mathrm{M}_\sharp$ such that $\mathrm{L(M)} = \mathrm{L}_\sharp$ is generated, we convert $\mathrm{M}_\sharp$ to a finite automaton M accepting L.
- This is a joint work with Azadeh Farzon, Yu-Fang Chen, Edmund M. Clarke, and Yih-Kuen Tsay.

Learning Minimal Assumptions I

- Recall the proof rule:

$$\frac{M_0 \| A \models P \qquad M_1 \models A}{M_0 \| M_1 \models P}$$

- Any assumption A such that

$$L(M_1) \subseteq L(A) \subseteq L(P) \cup \overline{L(M_0)}$$

  suffices.
- In practice, we would like A to have as few states as possible.
  - Otherwise, checking $M_0 \| A \models P$ may be more expensive than checking $M_0 \| M_1 \models P$.
  - Particularly, generating the weakest assumption does not necessarily alleviate the state explosion problem.

Learning Minimal Assumptions II

- Let J and K be regular languages.
- Consider a finite automaton S such that

$$J \subseteq L(S) \text{ and } L(S) \cap K = \emptyset.$$

- This is called a separating automaton for J and K.
- The minimal separating automaton for J and K is a separating automaton with the least number of states.
- Intuitively, J contains all the good behaviors and K contains all the bad behaviors.
- A separating automaton has all good behaviors and misses all bad behaviors.
- Finding the minimal separating automaton is a known problem in circuit synthesis.

Learning Minimal Assumptions III

- Now consider $J = L(M_1)$ and $K = \overline{L(P)} \cap L(M_0)$.
- A separating automaton A for J and K satisfies

$$L(M_1) \subseteq L(A) \text{ and } L(A) \cap (\overline{L(P)} \cap L(M_0)) = \emptyset.$$

- Note that $L(A) \cap (\overline{L(P)} \cap L(M_0)) = \emptyset$ iff $L(A) \subseteq \overline{(\overline{L(P)} \cap L(M_0))}$ iff $L(A) \subseteq L(P) \cup \overline{L(M_0)}$.
- Hence a minimal separating automaton for J and K is a minimal assumption.
- This is a joint work with Yu-Fang Chen Azadeh Farzan, Edmund M. Clarke, and Yih-Kuen Tsay.

Proof Rule Derivation I

- So far, we assume that the proof rule is fixed. We only consider the following rule:

$$\frac{M_0 \| A \models P \qquad M_1 \models A}{M_0 \| M_1 \models P}$$

- Of course, it is not hard to establish the soundness of this rule.
  - Assume $L(M_0) \cap L(A) \subseteq L(P)$ and $L(M_1) \subseteq L(A)$. We have $L(M_0) \cap L(M_1) \subseteq L(M_0) \cap L(A) \subseteq L(P)$.
- But there are other rules such as

$$\frac{M_0 \| A_0 \models P \qquad M_1 \| A_1 \models P \qquad \overline{A_0} \| \overline{A_1} \models P}{M_0 \| M_1 \models P}$$

- And circular rules such as

$$\frac{M_0 \| P_1 \models P_0 \qquad P_0 \| M_1 \models P_1}{M_0 \| M_1 \models P_0 \| P_1} \mathcal{C}$$

Proof Rule Derivation II

- By identifying classes of languages as models of classical or intuitionistic logic, we can establish these rules automatically.
- If you are interested, we can discuss it offline.

```
Coq < Goal forall M0 M1 A P : Prop,
Coq <      ((M0 /\ A -> P) /\ (M1 -> A)) ->
Coq <      (M0 /\ M1 -> P) .
1 subgoal

============================
   forall M0 M1 A P : Prop, (M0 /\ A -> P) /\ (M1 -> A) -> M0 /\ M1 -> P

Unnamed_thm < tauto .
Proof completed.
```

Conclusions

- For decades, the research community tried to devise contextual assumptions by inspecting intrinsic properties.
- These attempts fail because it is so hard to "guess" designers' intention.
- Algorithmic learning theory however does not posit to attain intelligence.
- Instead, it reformulates the problem such that mechanical solutions are possible.
- It is indeed useful in finding contextual assumptions.
  - Few people would believe it is possible a few years ago.

Relation with Program Analysis

- Can machine learning be applied to invariant generation?
  - If invariants are in propositional logic, it may be possible.
    - There is an algorithm for learning Boolean formulae.
  - For invariants in separation logic, it is unclear.
    - What about the fragment of symbolic heaps $\Pi|\Sigma$?
- Interpolations have been used rather often recently.
  - An interpolant I of A and B is a formula such that (1) $A \Rightarrow I$; and (2) $I \wedge B$ is a contradiction. Moreover, $\mathrm{var}(I) \subseteq \mathrm{var}(A) \cap \mathrm{var}(B)$.
  - Techniques for abstraction refinement have been developed.
  - Is there an interpolation theorem for (fragments of) separation logic?

Thank you!