

# DECENTRALIZED ALGORITHM FOR RANKING LEARNING

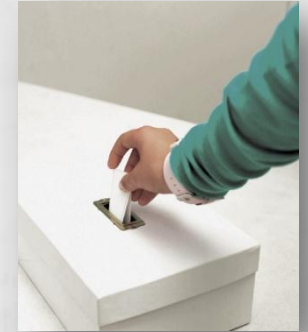
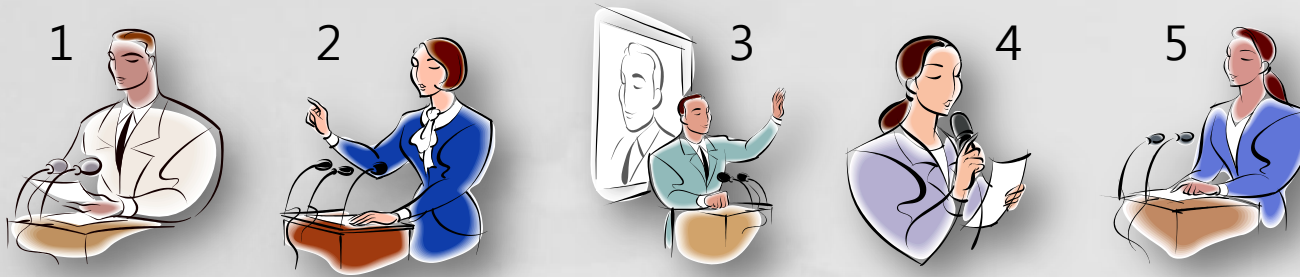
*ROSAEC center 2010 Workshop*

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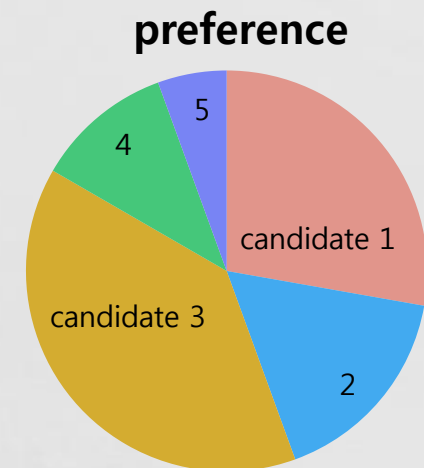
Kyomin Jung  
AALAB, KAIST

Milan Vojnovic  
Microsoft Research

# Decentralized Ranking Learning Problem

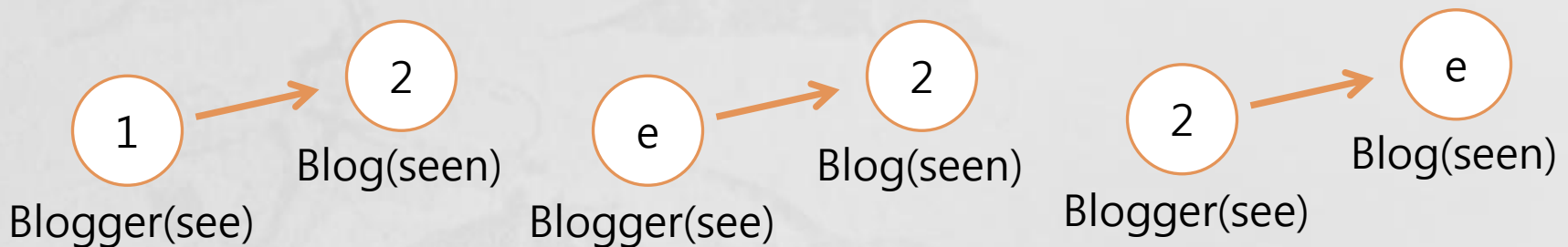


- (Presidential) election (or preference on fashion brands)
- Centralized/decentralized algorithm
- **Problem:** How to identify *the most frequent item*(candidate) by locally exchanging information between the nodes.
- **Local message exchange:** Blogger sees Blog around her.
- **Extended (General) version of this problem:** How to identify *k most frequent items in order*.



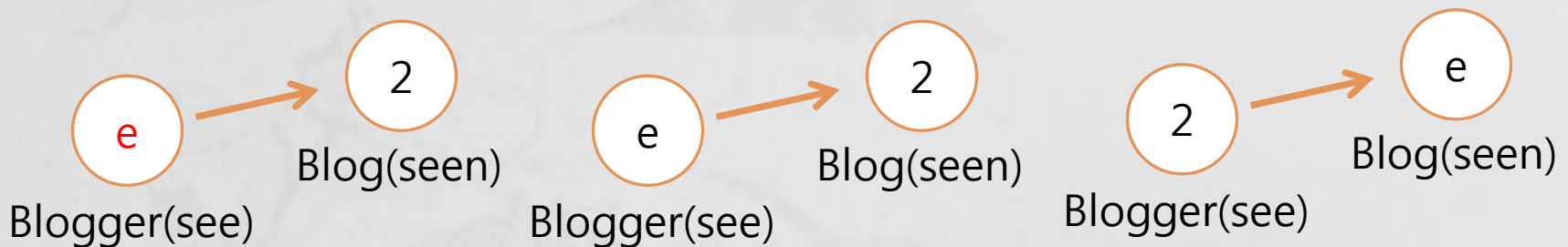
# Previous Works

- Classical voter model – two candidates,  $\{1,2\}$ .
- **Using Three States for Binary Consensus on Complete Graphs**, Infocom 2009 (Perron, Vasudevan, and Vojnovic)
  - Complete graph
  - Comparison between **ternary signaling & state** and **binary signaling & ternary state**
- **Convergence Speed of Binary Interval Consensus**, Infocom 2010 (Draief and Vojnovic)
  - Upper bound on the expected convergence time that holds for arbitrary connected graph e.g. Complete, star-shaped, Erdos-Renyi random graphs



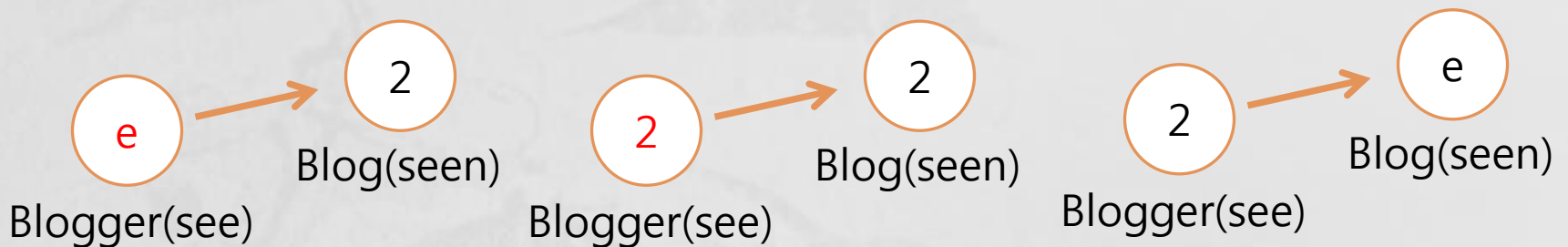
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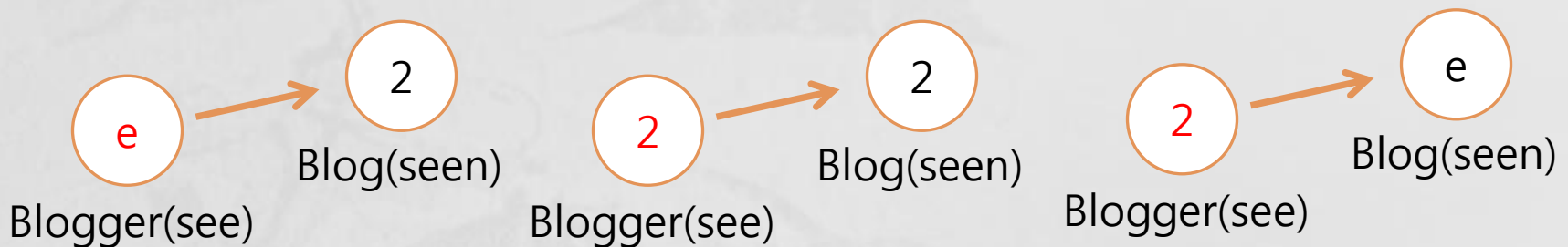
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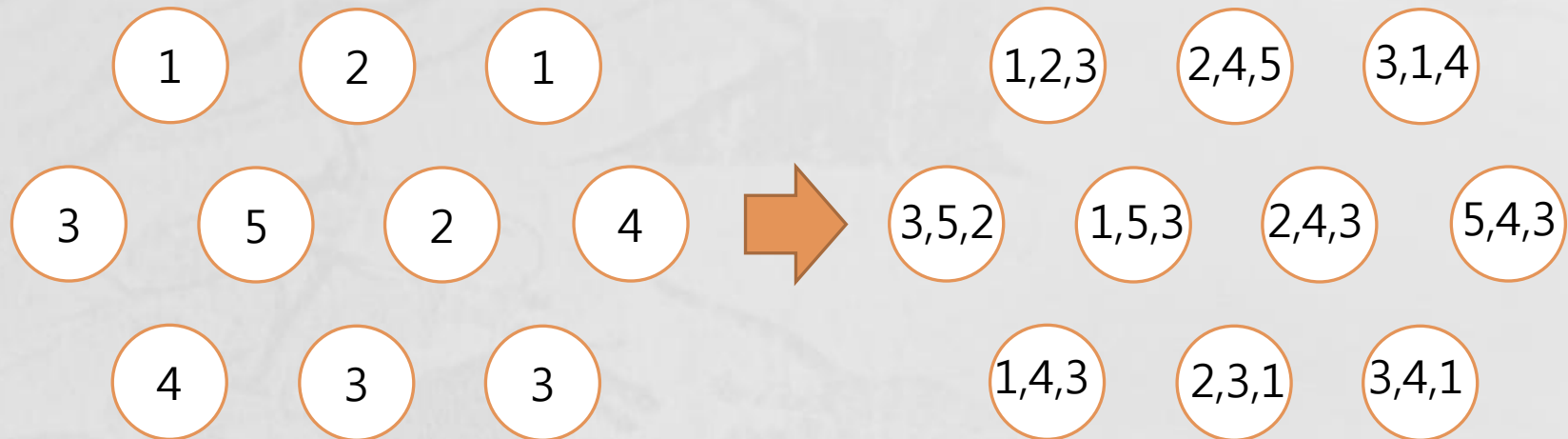
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# Ranking Learning

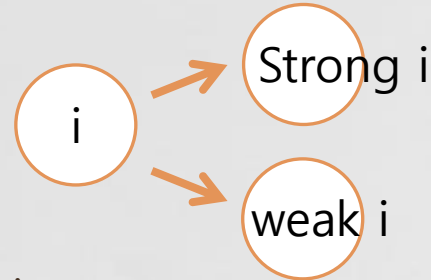
- **Finding the most frequent sorting state = Learning top-k ranking**
- **Sorting state assignment** - Using a rule, we assign a k-tuple as its new state instead of its initial single state for each node. → **Randomized Algorithm**
- **Deviation** between the random number and its expectation **is small**.

e.g.  $N=10$ ,  $M=5$ ,  $k=3$



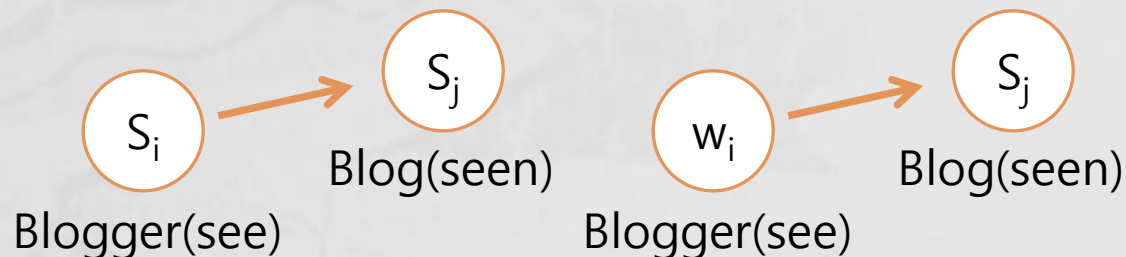
# Finding the Most Frequent Item

- Introduction of Strong state, weak state



- Distributed algorithm for the most frequent item

- If the instigator node is in a **strong state**, then it switches to the **corresponding weak state** if the encountered node holds a **different strong state**.
- If the instigator node is in a **weak state**, then it **copies the state of the encountered node**.

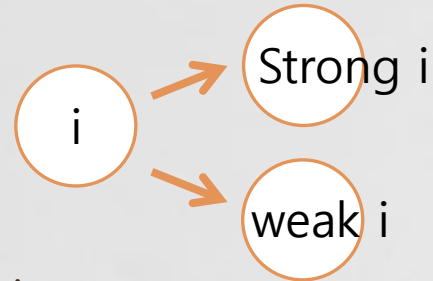


- We proved **convergence for complete graph theoretically**.
- We showed convergence for **random graphs** (Erdos-Renyi and Scale-free) **empirically**.

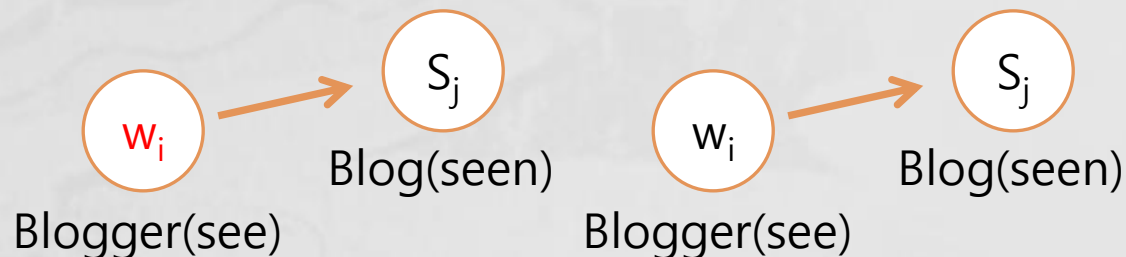


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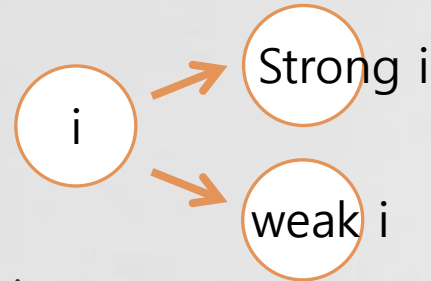
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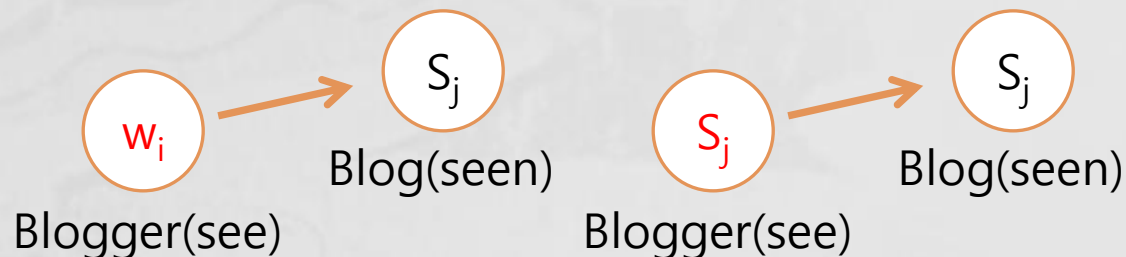
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# Future Works and Conclusion

- Conclusion
  - For  $m$ -candidate case, we developed a distributed algorithm that is parsimonious w.r.t the memory per node and information exchanged between the nodes for each node to identify the most frequent item.
  - Using the above algorithm, we learn the top- $k$  ranking (in order or w/o order).
- Future work
  - Study on theoretical convergence for other networks of interest, including random graphs (Erdos-Renyi and Scale-free).
  - Improvement on the speed and the probability of error.
  - Suggestion of distributed algorithm two-way communication case.