

<Lightening Talk>

# Robot Motion Programming

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# Formal Language

Alphabet  $\Sigma$

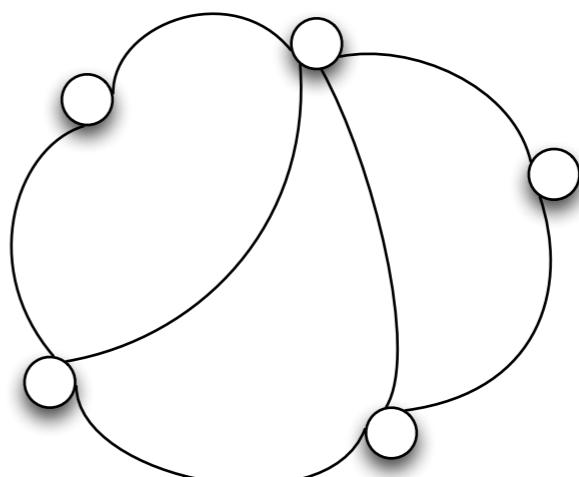
Language  $L \subseteq \Sigma^*$

FSM  $(\Sigma, X, Y, \delta, \gamma)$

$\Sigma, X, Y$ : finite sets

$$\delta : X \times \Sigma \rightarrow X$$

$$\gamma : X \rightarrow Y$$



$$x_{i+1} = \delta(x_i, u_i)$$

# Motion Description Language

Brockett [88,90]

**Alphabet**  $U$

**Language**  $L \subseteq U^*$

**Kinematic Machine**  $(U, X, Y, G, h)$

$U$  : control input space

$X$  : joint space

$Y$  : output space

$h : X \rightarrow Y$



$$\dot{x} = G(x)u$$

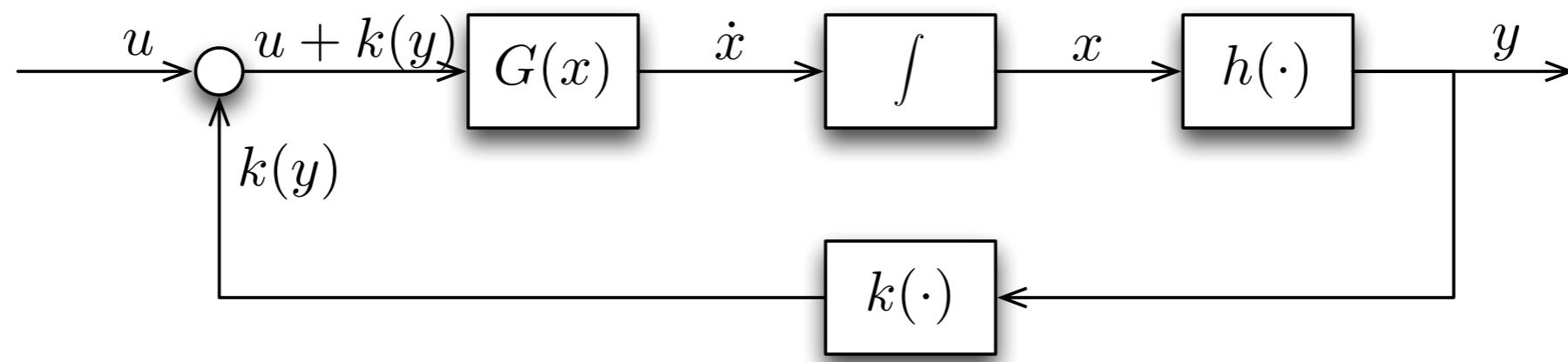
# Atom of Language

- $(u, k, T)$
- $$\dot{x} = G(x)u$$

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- $(u, k, T)$

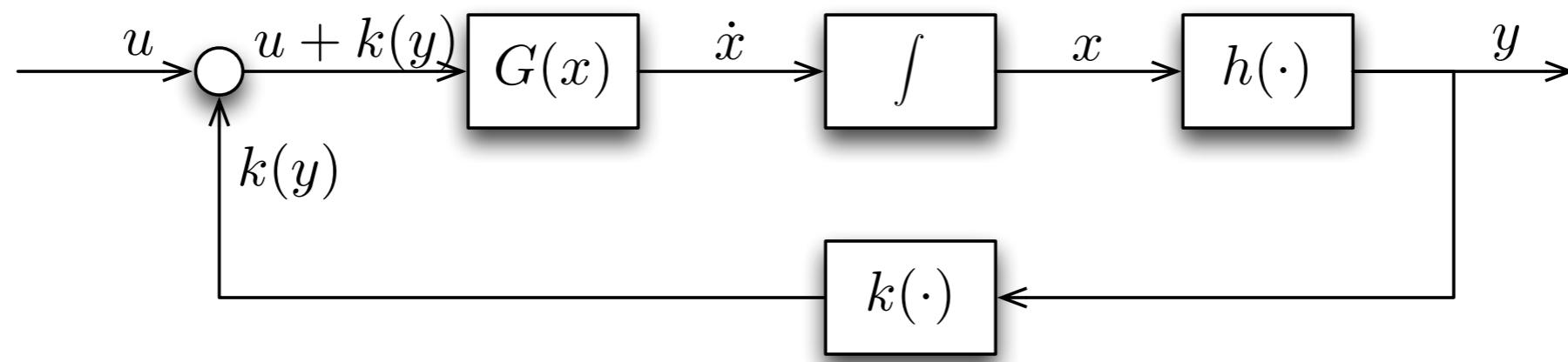
$$\dot{x} = G(x)(u + k(y)); \quad y = h(x); \quad 0 \leq t < T$$



# Atom of Language

- $(u, k, T)$

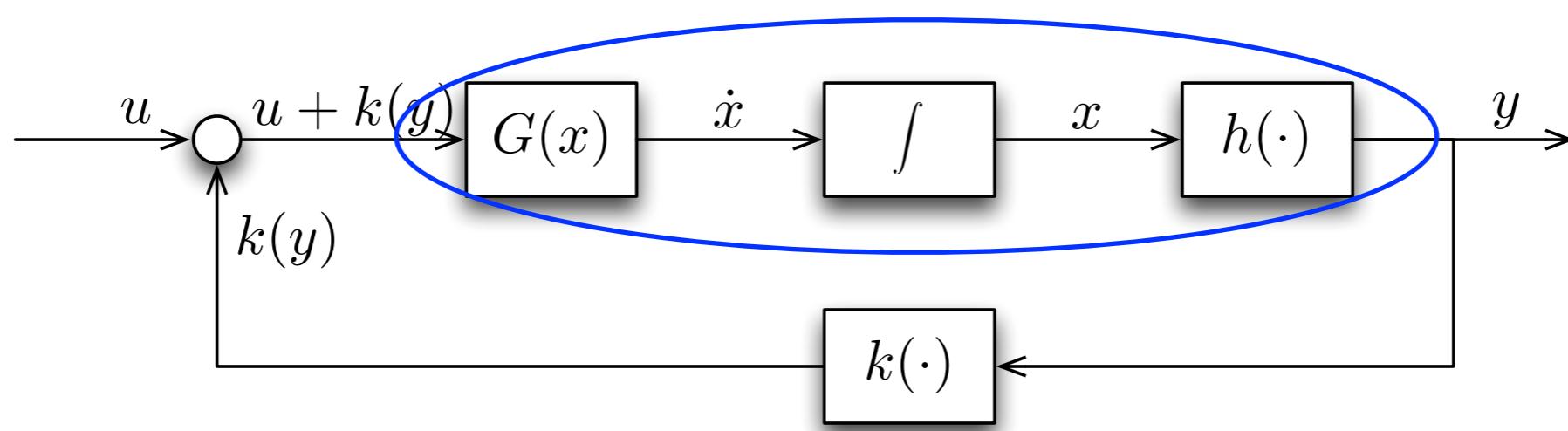
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# Atom of Language

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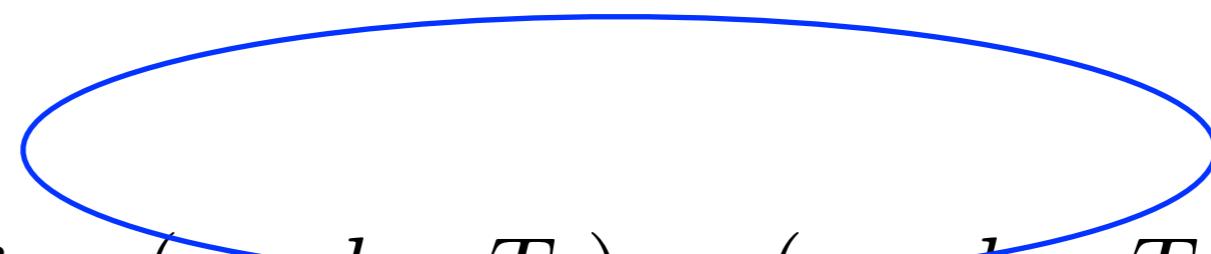
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# Atom of Language

- $(u, k, T)$

$$\dot{x} = G(x)(u + k(y)); \quad y = h(x); \quad 0 \leq t < T$$



- **input string**  $(u_1, k_1, T_1) \cdots (u_n, k_n, T_n)$

$$\dot{x} = G(x)(u_1 + k_1(y));$$

$$t_0 \leq t < t_0 + T_1$$

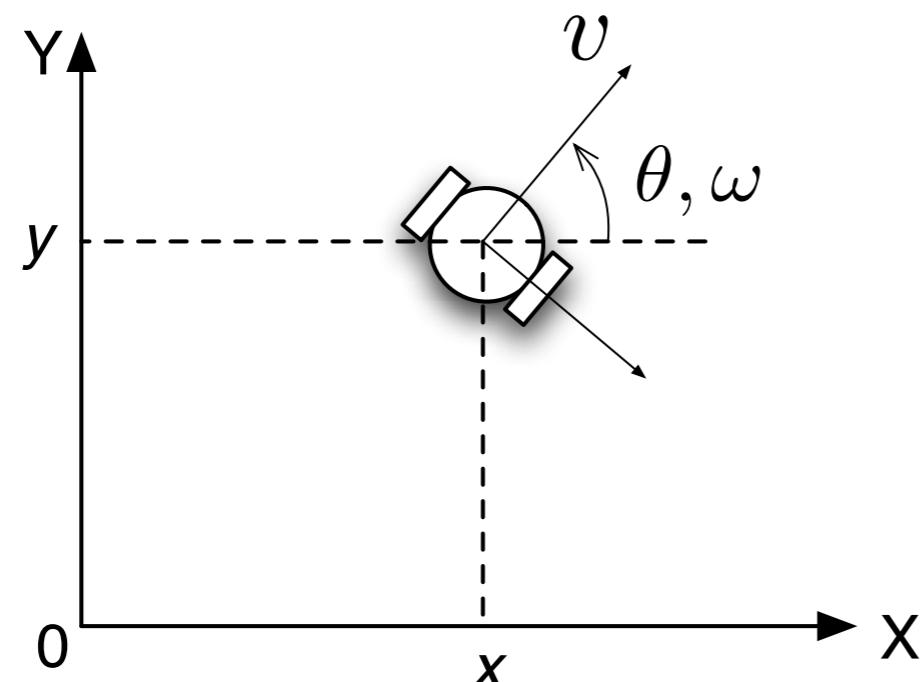
⋮

⋮

$$\dot{x} = G(x)(u_n + k_n(y)); \quad t_0 + T_1 + \cdots + T_{n-1} \leq t < t_0 + T_1 + \cdots + T_n$$

# Mobile Robot

- Governing Equation



$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$



$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t))\mathbf{u}(t)$$

$v$ : driving velocity

$\omega$ : steering velocity

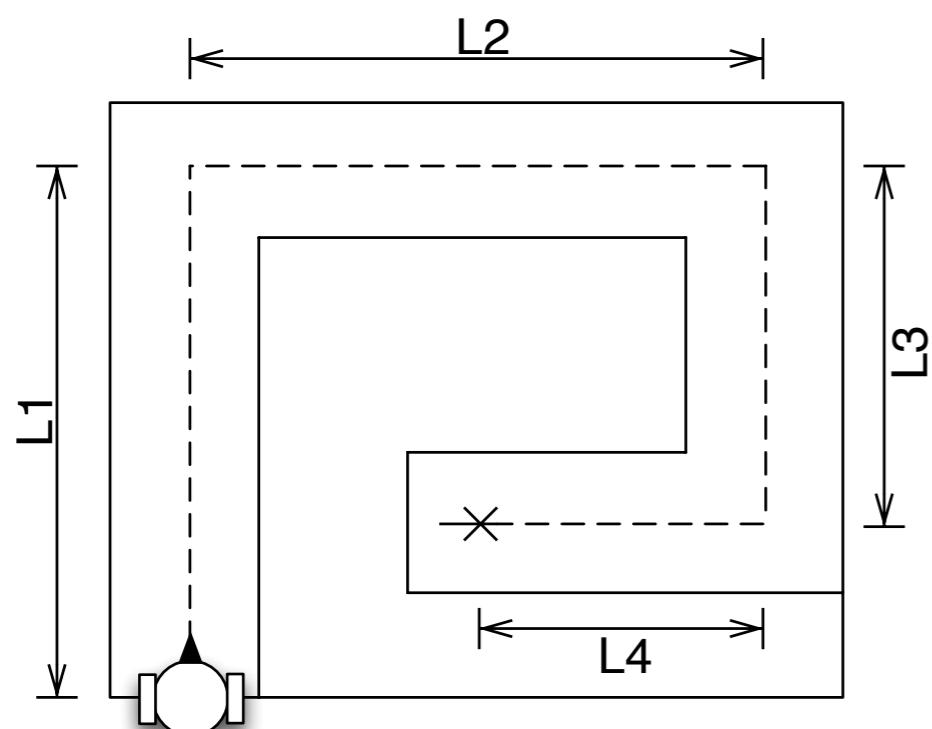
# Motion Description

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t))\mathbf{u}(t) \quad \text{for } t_0 \leq t < t_0 + T$$

$(\mathbf{u}, T)$	Description	Command
$((1, 0), d)$	move forward for $\mathbf{d}$ meters with 1 $m/s$	$mf(d)$
$((-1, 0), d)$	move backward for $\mathbf{d}$ meters with 1 $m/s$	$mb(d)$
$((0, 1), a)$	turn left for $\mathbf{a}$ radians with 1 $rad/s$	$tl(a)$
$((0, -1), a)$	turn right for $\mathbf{a}$ radians with 1 $rad/s$	$tr(a)$
$((0, 0), t)$	stop for $\mathbf{t}$ sec	$stop(t)$

# Maze

- Motion Programming



$mf(L1)$

$tr\left(\frac{\pi}{2}\right)$

$mf(L2)$

$tr\left(\frac{\pi}{2}\right)$

$mf(L3)$

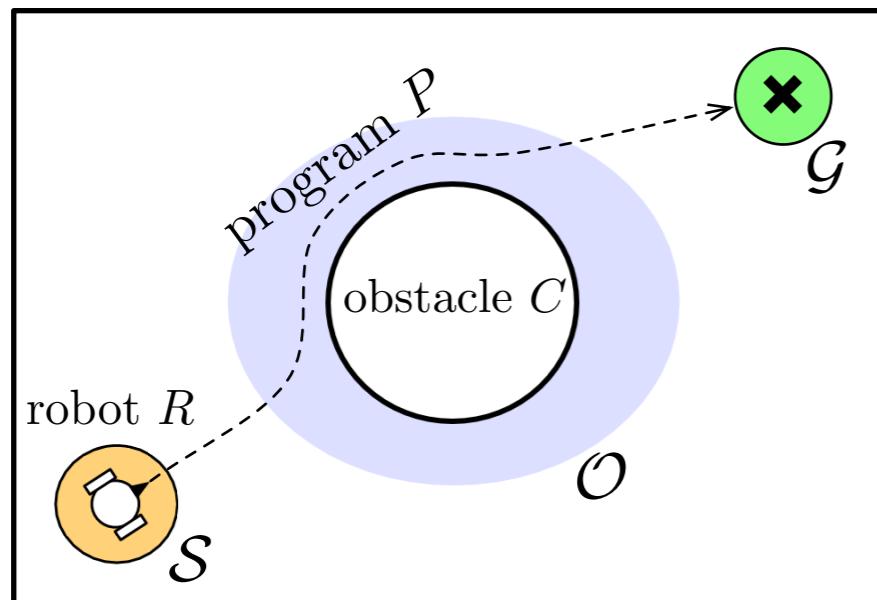
$tr\left(\frac{\pi}{2}\right)$

$mf(L4)$

$stop()$

# Static Analysis I

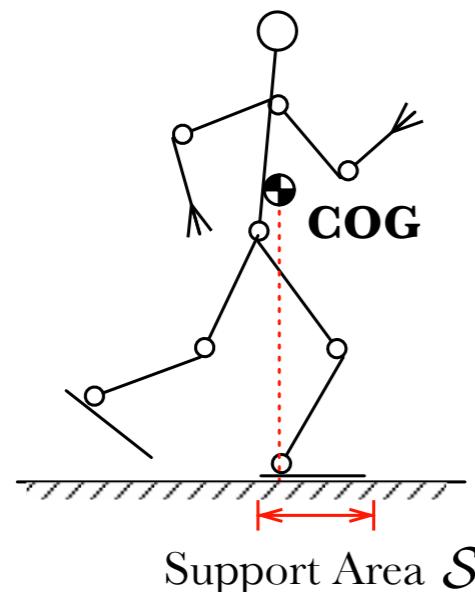
- Verifying path plan:



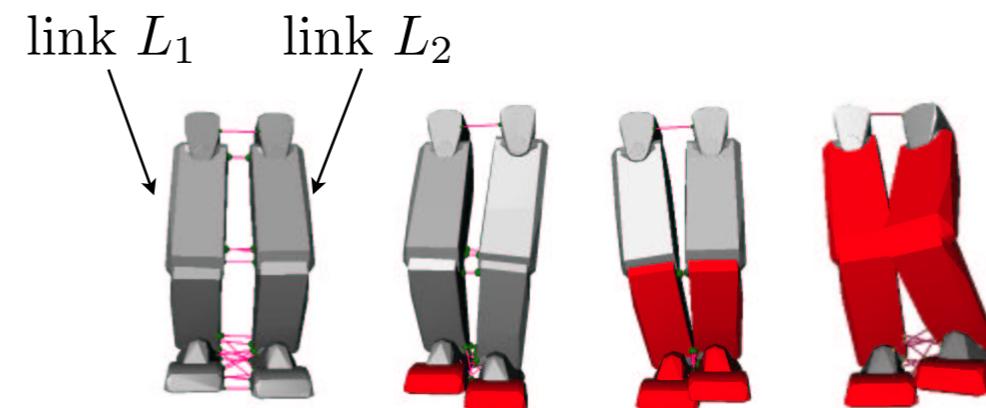
Given  $R_{initial} \in \mathcal{S}$  and  $C \in \mathcal{O}$ ,  
does motion program  $P$   
guarantee  $R_{final} \in \mathcal{G}$  ?

# Static Analysis II

- Balance
- Self-collision Avoidance



Does program  $P$  guarantee  
COG in  $S$ ?



Does program  $P$  guarantee that  
 $\forall p \in L_1$  and  $\forall q \in L_2$  are not  
coincided?