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Controller Design for UAV Formation Flight Using Consensus based Decentralized Approach

Joongbo Seo[1], Chaeik Ahn[2], Youdan Kim[3]

[1] School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea
(Tel : +82-2-880-1552; E-mail: sjb@snu.ac.kr)

[2] School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea
(Tel : +82-2-880-1907; E-mail: sety@snu.ac.kr)

[3] School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea
(Tel : +82-2-880-7398; E-mail: : ydkim@snu.ac.kr)

Abstract: A consensus-based feedback linearization method is proposed to maintain a specified time-varying geometric configuration for formation flight of multiple autonomous vehicles. In this approach, any explicit leader does not exist in the formation team, and therefore the proposed control strategy requires only the local neighbor-to-neighbor information between vehicles. The information flow topologies between the vehicles can be defined by Graph Laplacian matrix, and the formation flight can be achieved by the proposed feedback linearization with consensus algorithm. The stability analysis of the proposed controller is also performed. Numerical simulation is performed for the rotary type unmanned aerial vehicles to validate the performance of the proposed controller.

Keywords: Formation Flight, Consensus Protocol, Feedback Linearization, Cooperative Control, Graph Theory, Unmanned Aerial Vehicle.

1. INTRODUCTION

Recently, many researches on the formation flight control for multiple Unmanned Aerial Vehicles(UAVs) have been performed because various missions can be successfully completed by the formation flight. Most of the formation flight researches are performed on the leader-follower approach, where some vehicles are designated as leaders while others are designated as followers. In this approach, the leader tracks the predefined trajectory, and the follower tracks the nearest leaders according to given schemes. It is easy to analyze and implement the leader-follower controller. However, there are some limitation, for example the leader is a single point of failure for the formation, and therefore this approach is not robust with respect to the UAV failure.

Consensus type problems have been studied for the cooperative control of mobile autonomous agents, where each agent in a team updates its information state based on the information states of its local neighbors. Critical problem for cooperative control is to design appropriate protocols and algorithms. The group of vehicles can converge to a consistent view of the shared information in the presence of limited and unreliable information exchange and dynamically changing interaction topologies. In this paper, a leaderless formation control strategy based on the consensus algorithms is proposed. The stability of proposed control strategy is analyzed, and numerical simulation is performed to verify the performance of the proposed formation flight controller.

2. FORMATION FLIGHT OF MULTIPLE UAVS

Feedback linearization

Let us consider a following nonlinear UAV model for formation flight. [1]

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\psi}_i\\ \dot{v}_i\\ \dot{w}_i\end{bmatrix} = \begin{bmatrix} v_i\cos\psi_i \\ v_i\sin\psi_i \\ \omega_i \\ -\dfrac{v_i}{\alpha_{vi}} \\ -\dfrac{r_i}{\alpha_{wi}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{1}{\alpha_{vi}} & 0 \\ 0 & \dfrac{1}{\alpha_{ri}} \end{bmatrix} \begin{bmatrix} v_i^c \\ w_i^c \end{bmatrix} \tag{1}$$

where state vector $X_i = [x \;\; y \;\; \psi \;\; v \;\; w]^T$, control inputs vector $U_i = [v_i^c \;\; w_i^c]^T$, and v_i is a forward velocity, w_i is a angular velocity, (x_i, y_i) is a two dimensional inertial position, and ψ_i is a heading angle. Input variable v_i^c is a commanded forward velocity, and r_i^c is a commanded heading rate. Note that aircraft dynamics are model as the first order time delay model with positive time constants of α_{vi} and α_{wi}.

Equations (1) can be rewritten as

$$\dot{X}_i = f(X_i) + g_i U_i \tag{2}$$

where $X_i = (x_i, y_i, \psi_i, v_i, w_i)$, $U_i = (v_i^c, w_i^c)$.

To avoid the non-holonomic constraint, let us define

$$r_{fi} = r_i + d_i \begin{bmatrix}\cos\psi_i \\ \sin\psi_i\end{bmatrix} \tag{3}$$

where $r_i = (x_i, y_i)^T$, and $r_{fi} = (x_{fi}, y_{fi})^T$. Note that x_i and y_i represent i-th UAV's lateral CG position in the inertial coordinates, x_{fi} and y_{fi} represent the inertial position of a f_i located at a distance d_i along the x-body axis of the i-th UAV, presuming zero pitch angle. The coordination of (x_{fi}, y_{fi}) will be used instead of (x_i, y_i) to simplify the design of the coordination algorithms.

Differentiating Eq.(3) with respect to time twice gives

$$\ddot{r}_{fi} = \begin{bmatrix} -\dfrac{v_i}{\alpha_{vi}}\cos\psi_i + \dfrac{d_i}{\alpha_{wi}}\omega_i\sin\psi_i - v_i\omega_i\sin\psi_i - d_i\cos\psi_i\omega_i^2 \\ -\dfrac{v_i}{\alpha_{vi}}\sin\psi_i + \dfrac{d_i}{\alpha_{wi}}\omega_i\cos\psi_i + v_i\omega_i\cos\psi_i - d_i\sin\psi_i\omega_i^2 \end{bmatrix} + \begin{bmatrix} \dfrac{\cos\psi_i}{\alpha_{vi}} & -\dfrac{d_i\sin\psi_i}{\alpha_{wi}} \\ \dfrac{\sin\psi_i}{\alpha_{vi}} & \dfrac{d_i}{\alpha_i}\cos\psi_i \end{bmatrix} \begin{bmatrix} v_i^c \\ w_i^c \end{bmatrix} \tag{4}$$

The above system, Eq.(2) and (3), has constant relative degree 2, and therefore the given system can be output feedback linearized. For feedback linearization, let us define the map $P: \mathbb{R}^5 \rightarrow \mathbb{R}^5$ as

$$\zeta_i = P(X_i) \triangleq \begin{bmatrix} r_{fi} \\ \dot{r}_{fi} \\ \psi_i \end{bmatrix} = \begin{bmatrix} x_i + d_i\cos\psi_i \\ y_i + d_i\sin\psi_i \\ v_i\cos\psi_i - d_i\omega_i\sin\psi_i \\ v_i\sin\psi_i + d_i\omega_i\cos\psi_i \\ \psi_i \end{bmatrix} \tag{5}$$

The above map is a diffeomorphism, and its inverse can be written as

$$X_i = P_i^{-1}(\zeta_i) = \begin{bmatrix} \zeta_{1i} - d_i\cos\zeta_{5i} \\ \zeta_{2i} - d_i\sin\zeta_{5i} \\ \zeta_{5i} \\ \dfrac{1}{2}\zeta_{3i}\cos\zeta_{5i} + \dfrac{1}{2}\zeta_{4i}\sin\zeta_{5i} \\ -\dfrac{1}{2d_i}\zeta_{3i}\sin\zeta_{5i} + \dfrac{1}{2d_i}\zeta_{4i}\cos\zeta_{5i} \end{bmatrix} \tag{6}$$

Using the transformed coordinates, Eqs. (2)-(3) can be written as

$$\begin{bmatrix} \dot{\zeta}_{1i} \\ \dot{\zeta}_{2i} \end{bmatrix} = \begin{bmatrix} \zeta_{3i} \\ \zeta_{4i} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\zeta}_{3i} \\ \dot{\zeta}_{4i} \end{bmatrix} = \begin{bmatrix} -\frac{v_i}{\alpha_{vi}}\cos\psi_i + \frac{d_i}{\alpha_{wi}}\omega_i \sin\psi_i - v_i\omega_i \sin\psi_i - d_i \cos\psi_i \omega_i^2 \\ -\frac{v_i}{\alpha_{vi}}\sin\psi_i + \frac{d_i}{\alpha_{wi}}\omega_i \cos\psi_i + v_i\omega_i \cos\psi_i - d_i \sin\psi_i \omega_i^2 \end{bmatrix} + \begin{bmatrix} \frac{\cos\psi_i}{\alpha_{vi}} & -\frac{d_i \sin\psi_i}{\alpha_{wi}} \\ \frac{\sin\psi_i}{\alpha_{vi}} & \frac{d_i}{\alpha_{vi}}\cos\psi_i \end{bmatrix} \begin{bmatrix} v_i^c \\ w_i^c \end{bmatrix} \tag{7}$$

$$\dot{\zeta}_{5i} = -\frac{1}{2d_i}\zeta_{3i}\sin\zeta_{5i} + \frac{1}{2d_i}\zeta_{4i}\cos\zeta_{5i}$$

Now, the output feedback linearizing control input can be given by

$$u_i = \begin{bmatrix} v_i^c \\ w_i^c \end{bmatrix} = \begin{bmatrix} \frac{\cos\psi_i}{\alpha_{vi}} & -\frac{d_i \sin\psi_i}{\alpha_{wi}} \\ \frac{\sin\psi_i}{\alpha_{vi}} & \frac{d_i}{\alpha_{wi}}\cos\psi_i \end{bmatrix}^{-1} \left[\mu_i - \begin{pmatrix} -\frac{v_i}{\alpha_{vi}}\cos\psi_i + \frac{d_i}{\alpha_{wi}}\omega_i \sin\psi_i - v_i\omega_i \sin\psi_i - d_i \cos\psi_i \omega_i^2 \\ -\frac{v_i}{\alpha_{vi}}\sin\psi_i + \frac{d_i}{\alpha_{wi}}\omega_i \cos\psi_i + v_i\omega_i \cos\psi_i - d_i \sin\psi_i \omega_i^2 \end{pmatrix} \right] \tag{8}$$

where μ_i is the additional control input which can be designed for formation flight. Note that the inverse in Eq.(8) always exists because the matrix is nonsingular. The above control input will be used in the consensus protocol which will be explained in detail in the subsequent section.

Using Eq.(9) in Eq.(8), we have

$$\begin{bmatrix} \dot{\zeta}_{1i} \\ \dot{\zeta}_{2i} \end{bmatrix} = \begin{bmatrix} \zeta_{3i} \\ \zeta_{4i} \end{bmatrix} = v \tag{9}$$

$$\begin{bmatrix} \dot{\zeta}_{3i} \\ \dot{\zeta}_{4i} \end{bmatrix} = \dot{v} = \mu_i$$

Note that the ζ_{5i} dynamics of Eq. (7) is related to the internal dynamics, which are rendered unobservable and uncontrollable by the transformation (5). The zero dynamics are found by setting $\zeta_{1i} = \zeta_{2i} = \zeta_{3i} = \zeta_{4i} = 0$, then $\dot{\zeta}_{5i} = 0$. Therefore, zero dynamics are stable but not asymptotically stable. Because $\zeta_{5i} = \psi_i$ and (ζ_{3i}, ζ_{4i}) represent the velocity, the angle ψ_i will stop changing when the position of UAV stops moving. Therefore, the following equations of motion can be obtained.

$$\dot{r}_{fi} = v_{fi}, \qquad \dot{v}_{fi} = \mu_{fi} \tag{11}$$

where $r_{fi} = \left[x_{fi}, y_{fi}\right]^T$, $v_{fi} = \left[\dot{x}_{fi}, \dot{y}_{fi}\right]^T$ and $\mu_{fi} = \left[\mu_{xi}, \mu_{yi}\right]^T$.

Now, control law can be applied to design μ_{fi} such that multiple UAVs can fly with a pre-defined formation velocity given by $v_F^d(t)$. Then the team can preserve a specified geometric configuration during the flight.

CONTROL USING CONSENSUS STRATEGY AND CONVERGENCE ANALYSIS

Let us consider the vehicle dynamics of each UAV as

$$\begin{aligned} \dot{r}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \tag{12}$$

where $r_i \in \mathbb{R}^m$ and $v_i \in \mathbb{R}^m$ denote the position and velocity of i-th vehicle, respectively. Figure 1 shows the geometry of formation considered in this study. The position of i-th node can be represented as

$$r_i = r_{oi} + r_{iF} \tag{13}$$

where r_{oi} denotes the position of formation center, and r_{iF} denotes a vector from the formation center to the i-th node.

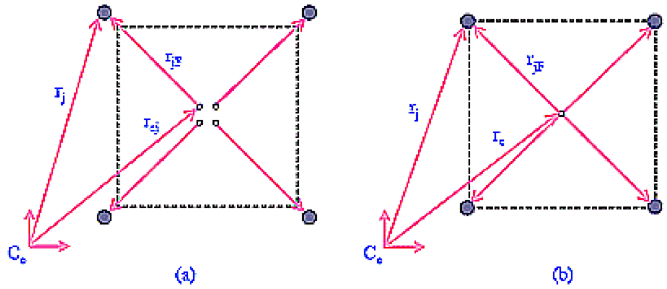

Fig. 1 Consensus reached on a formation center

In this study, let us choose the control input of the i-th vehicle in Eqs. (11) and (12) as

$$u_i = \ddot{r}_{iF} + \dot{v}_F^d - \alpha(v_i - \dot{r}_{iF} - v_F^d) - \beta(r_i - r_{iF} - \int v_F^d\, dt) - \sum_{j=1}^{n} g_{ij}k\{[(r_i - r_{iF}) - (r_j - r_{jF})] + \gamma[(v_i - \dot{r}_{iF}) - (v_j - \dot{r}_{jF})]\} \tag{14}$$

where $v_F{}^d \in \mathbb{R}^m$ specifies the nominal formation velocity, $\alpha,\ \beta,\ \gamma$ are positive constants, and g_{ij} and k are determined by the Laplacian graph.

Now, let us analyze the convergence property of the control law. For simplicity, let us introduce the following variable $z_i \in \mathbb{R}$.

$$z_i = r_i - \int v_F^d\, dt \tag{15}$$

$$\dot{z}_i = \dot{r}_i - v_F^d \tag{16}$$

Then, we have

$$\ddot{r}_i = u_i \tag{17}$$

$$\dot{z}_i = \ddot{r}_i - \dot{v}_F^d = u_i - \dot{v}_F^d \tag{18}$$

Substitution of Eqs. (14)-(17) into Eq.(18), we have

$$\dot{z}_i = \ddot{r}_{iF} - \alpha(\dot{z}_i - \dot{r}_{iF}) - \beta(\dot{z}_i - \dot{r}_{iF}) - \sum_{j=1}^{n} g_{ij}k\{[z_i - r_{iF}] - [z_j - r_{jF})] + \gamma[(\dot{z}_i - \dot{r}_{iF}) - (\dot{z}_j - \dot{r}_{jF})]\} \tag{19}$$

Or,

$$(\ddot{z}_i - \ddot{r}_{iF}) + \alpha(\dot{z}_i - \dot{r}_{iF}) + \beta(\dot{z}_i - \dot{r}_{iF}) + \sum_{j=1}^{n} g_{ij}k\{[z_i - r_{iF}] - [z_j - r_{jF})] + \gamma[(\dot{z}_i - \dot{r}_{iF}) - (\dot{z}_j - \dot{r}_{jF})]\} = 0 \tag{20}$$

Let us define a new variable as

$$z_i - r_{iF} = e_i \tag{21}$$

Then, Eq.(20) can be simplified as

$$\ddot{e}_i + \alpha\dot{e}_i + \beta e_i + \sum_{j=1}^{n} g_{ij}k\{(e_i - e_j) + \gamma(\dot{e}_i - \dot{e}_j)\} = 0 \tag{22}$$

The objective of consensus protocol is to guarantee that $e_i \to 0$ and $e_j \to 0$ as $t \to \infty$, This means that $\xi_i - r_{iF} \to 0$ and $\dot{\xi}_i - \dot{r}_{iF} \to 0$ as $t \to \infty$, or $r_{oi} - \int v_F^d\, dt \to 0$ and $\dot{r}_{oi} - v_F^d \to 0$. Then, $r_{oi} \to r_{oj}$ and $\dot{r}_{oi} \to \dot{r}_{oj}$, which means the formation flight can be achieved.

Equation (22) can be written as

$$\frac{d}{dt}\{T\} = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ -\alpha I_n & -\beta I_n \end{bmatrix} T + \begin{bmatrix} 0_{n\times n} & I_n \\ -L & -\gamma L \end{bmatrix} T = \begin{bmatrix} 0_{n\times n} & I_n \\ -\alpha I_n - L & -\beta I_n - \gamma L \end{bmatrix} T \tag{23}$$

where $T \triangleq [e_1 \; e_2 \ldots e_n \; \dot{e}_1 \; \dot{e}_2 \ldots \dot{e}_n]^T$

$$\Gamma = \begin{bmatrix} 0_{n\times n} & I_n \\ -\alpha I_n - L & -\beta I_n - \gamma L \end{bmatrix} \tag{24}$$

Now, α, β, γ, L can be chosen properly such that all eigenvalues of matrix Γ are placed on the left-half plane. Note that α, β, γ are positive real constants, and graph Laplacian matrix L is a positive definite matrix as

$$L \triangleq [l_{ij}] \;,\; l_{ii} = k \;\text{ and }\; l_{ij} = -g_{ij}k \;,\; \forall i \neq j \tag{25}$$

where $k > 0$, $g_{ii} \triangleq 0$, $g_{ij} = 1$ if information flows from vehicle j to vehicle i , and $g_{ij} = 0$ otherwise, $\forall i \neq j$. In this study, six UAVs are considered. The information exchange topologies between the six agents are shown in Fig. 2, where an edge from the i-th agent to the j-th agent means that the j-th agent can receive information from the i-th one. Taking into account the measurements from sensors with limited fields of views or random communication data loss, an undirectional or directional information flow topology can be considered. Note that Fig. 2 has a balanced undirected path within Graph theory.

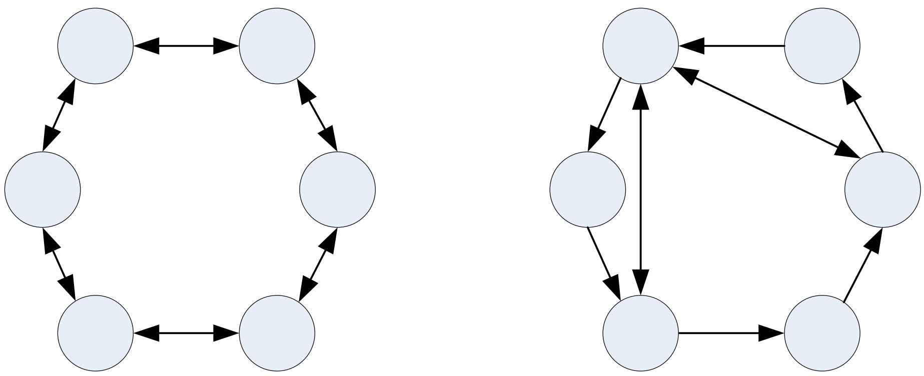

Fig. 2 Information exchange topology bet. six agents.

3. STABILITY ANALYSIS OF CONSENSUS PROTOCOL

Graph Laplacian matrix L can be determined by the method explained in the previous section, considering the given information flow. Given L, the information of its own we have, and at least, one of another vehicle are available. Therefore, it is possible to set the value of α, β, γ using matrix L. The eigenvalues of Γ can be computed as

$$\det(\lambda I_{2n} - \Gamma) = \det\left(\begin{bmatrix} \lambda I_n & -I_n \\ \alpha I_n + L & \lambda I_n + \beta I_n + \gamma L \end{bmatrix}\right) = \det\left((\lambda^2 + \lambda\beta + \alpha) I_n + (\lambda\gamma + 1) L\right) \tag{26}$$

Note that

$$\det(\lambda I_n + L) = \prod_{i=1}^{n} (\lambda - \mu_i) \tag{27}$$

where μ_i is the i-th eigenvalue of matrix $-L$. By comparing Eq.(26) and Eq.(27), we have

$$\det\left((\lambda^2 + \lambda\beta + \alpha) I_n + (\lambda\gamma + 1) L\right) = \prod_{i=1}^{n} \left[(\lambda^2 + \lambda\beta + \alpha) - (\lambda\gamma + 1)\mu_i\right] = 0 \tag{28}$$

Therefore, the eigenvalues of (26) can be obtained by solving the following equation.

$$(\lambda^2 + \lambda\beta + \alpha) - (\lambda\gamma + 1)\mu_i = 0 \tag{29}$$

or,

$$\lambda_{i\pm} = \frac{(\gamma\mu_i - \beta) \pm \sqrt{(\gamma\mu_i - \beta)^2 - 4(\alpha - \mu_i)}}{2} \tag{30}$$

Note that, there are two possible cases. To be positive definite matrix Γ, parameters $\alpha, \beta,$ and γ should be satisfied the following condition.

Case I) $(\gamma\mu_i - \beta)^2 - 4(\alpha - \mu_i) > 0$

$$(\gamma\mu_i - \beta) \pm \sqrt{(\gamma\mu_i - \beta)^2 - 4(\alpha - \mu_i)} < 0 \qquad i = 1..n \tag{31}$$

Case II) $(\gamma\mu_i - \beta)^2 - 4(\alpha - \mu_i) \le 0$

$$\gamma\mu_i - \beta < 0 \tag{32}$$

4. NUMERICAL SIMULATION

Figure 3 shows two different networks each vehicle is all strongly connected and balanced with the adjacent vehicle, and all vehicles can send and receive the information with assumption of half duplex transmission. Both network case (a) and case (b) are preserved with the regular triangle geometry. In this study, it is assumed that the communications between UAV5 and UAV4, UAV4 and UAV3 are disconnected. Then topology of case (a) should be changed to that of case (b). The parameter values used in the simulation are summarized in Table 1.

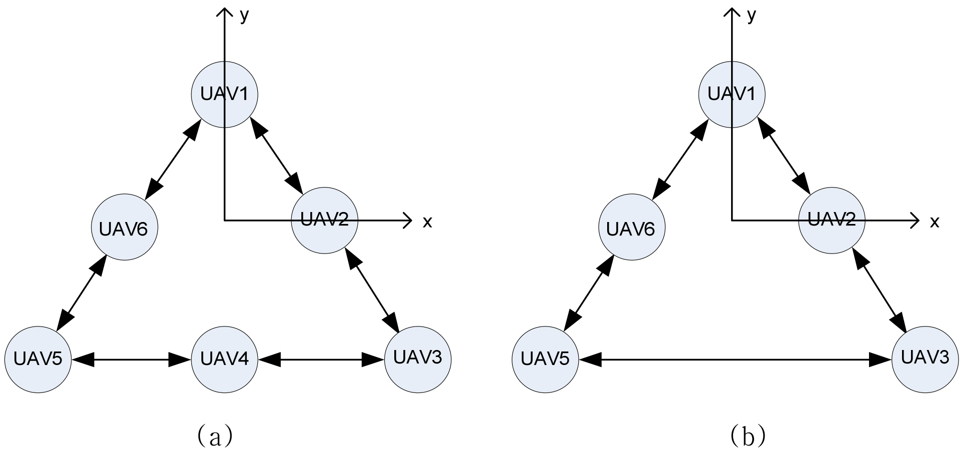

Fig. 3 Information exchange topology and geometry coordination

Parameter	Value
α_{vi}	1
α_{ri}	1
k_{ij}	0.5
v_i^C	$v_i^C \in [-5,5]\ m/s$
r_i^C	$r_i^C \in [-1,1]\ rad/s$
v_F^d	$2*[\sin(\theta_1(t)), \cos(\theta_1(t))]^T$
r_{1F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [0 \quad 7\sqrt{3}]^T$
r_{2F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [7 \quad 0]^T$
r_{3F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [14 \quad -7\sqrt{3}]^T$
r_{4F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [0 \quad -7\sqrt{3}]^T$
r_{5F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [-14 \quad -7\sqrt{3}]^T$
r_{6F}	$E(t)\ T_3(\theta_3(t))T_2(\theta_2(t))\ T_1(\theta_1(t))\ [-7 \quad 0]^T$

Table 1 Parameter values used in Simulation

In table 1 $T_i(t)$ and $E(t)$ denote the magnification and rotation matrix of formation flight as follow

$$E(t) = \begin{cases} 1, & t<90\sec \\ \frac{t-90}{30}+1, & 90 \le t < 100\sec \\ \frac{4}{3}, & t \ge 100\sec \end{cases} \qquad T_1(t) = \begin{cases} 0, & t<50\sec \\ \frac{t-50}{15}, & 50 \le t < 50+\frac{15\pi}{2}\sec \\ \frac{\pi}{2}, & t \ge 50+\frac{15\pi}{2}\sec \end{cases}$$

$$T_2(t) = \begin{cases} 0, & t<110\sec \\ \frac{t-110}{15}, & 110 \le t < 110+\frac{15\pi}{2}\sec \\ \frac{\pi}{2}, & t \ge 110+\frac{15\pi}{2}\sec \end{cases} \qquad T_3(t) = \begin{cases} 0, & t<150\sec \\ \frac{t-150}{15}, & 100 \le t < 150+\frac{15\pi}{4}\sec \\ \frac{\pi}{4}, & t \ge 150+\frac{15\pi}{4}\sec \end{cases}$$

$$T_4(t) = \begin{cases} 0, & t<185\sec \\ \frac{t-185}{15}, & 185 \le t < 185+\frac{15\pi}{4}\sec \\ \frac{\pi}{4}, & t \ge 185+\frac{15\pi}{4}\sec \end{cases}$$

$$T_2(\theta_2(t)) = \begin{bmatrix} \cos(\theta_2(t)) & \sin(\theta_2(t)) \\ -\sin(\theta_2(t)) & \cos(\theta_2(t)) \end{bmatrix}, \quad T_1(\theta_1(t)) = \begin{bmatrix} \cos(\theta_1(t)) & \sin(\theta_1(t)) \\ -\sin(\theta_1(t)) & \cos(\theta_1(t)) \end{bmatrix}$$

$$T_4(\theta_4(t)) = \begin{bmatrix} \cos(\theta_4(t)) & \sin(\theta_4(t)) \\ -\sin(\theta_4(t)) & \cos(\theta_4(t)) \end{bmatrix}, \quad T_3(\theta_3(t)) = \begin{bmatrix} \cos(\theta_3(t)) & \sin(\theta_3(t)) \\ -\sin(\theta_3(t)) & \cos(\theta_3(t)) \end{bmatrix}$$

For this particular example, the desired eigenvalues of matrix Γ are assigned as $-0.5 \pm 1.3i$, $-1 \pm 1i$, $-0.6 \pm 0.5i$, and $-0.5 \pm 0.9i$. Since four equations with three variables (α, β, γ) are considered in this study, a least square solution is obtained.

There exists the communication disconnect of UAV 4 at t=85sec, and the size of the desired triangular geometric configuration between six UAVs will be expanded 33 %, during t= [90; 100) seconds as shown by the definitions of E(t). Also, four rotation are performed at t = 50 seconds, t = 110 seconds t = 150 seconds and t = 185 seconds as shown by the definitions of $T_1(\theta_1(t))$, $T_2(\theta_2(t))$, $T_3(\theta_3(t))$ and $T_4(\theta_4(t))$. UAV is arranged initially in a hexagon which is summarized in Table 2.

UAV1	$[0 \quad 5]$
UAV2	$[5/\sqrt{3} \quad 5/2]$
UAV3	$[5/\sqrt{3} \quad -5/2]$
UAV4	$[0 \quad -5]$
UAV5	$[-5/\sqrt{3} \quad -5/2]$
UAV6	$[-5/\sqrt{3} \quad 5/2]$

Table 2 Initial Point of each UAVs

Figures 4- 6 show the simulation result. Figure 4 and 5 show that the formation is preserved with the desired time-varying triangular formation, and each UAV flies with a nominal formation given velocity. Note that UAV3 show tries to move for keeping relative velocity to the others at the rotation corners. Right after communication disconnect of UAV4, UAV3 and UAV5 begin to have a new network to maintain a formation. Figure 5 shows the values of distance between 6 vehicles. Distances between each UAV has the similar value, 15(m) and 20(m). During the expansion errors are within 2.4(m) error range, and the distance between UAVs are almost constant during the whole flight Control input responses are shown in Figure 5. Each control input of UAV is acceptable within the limited range.

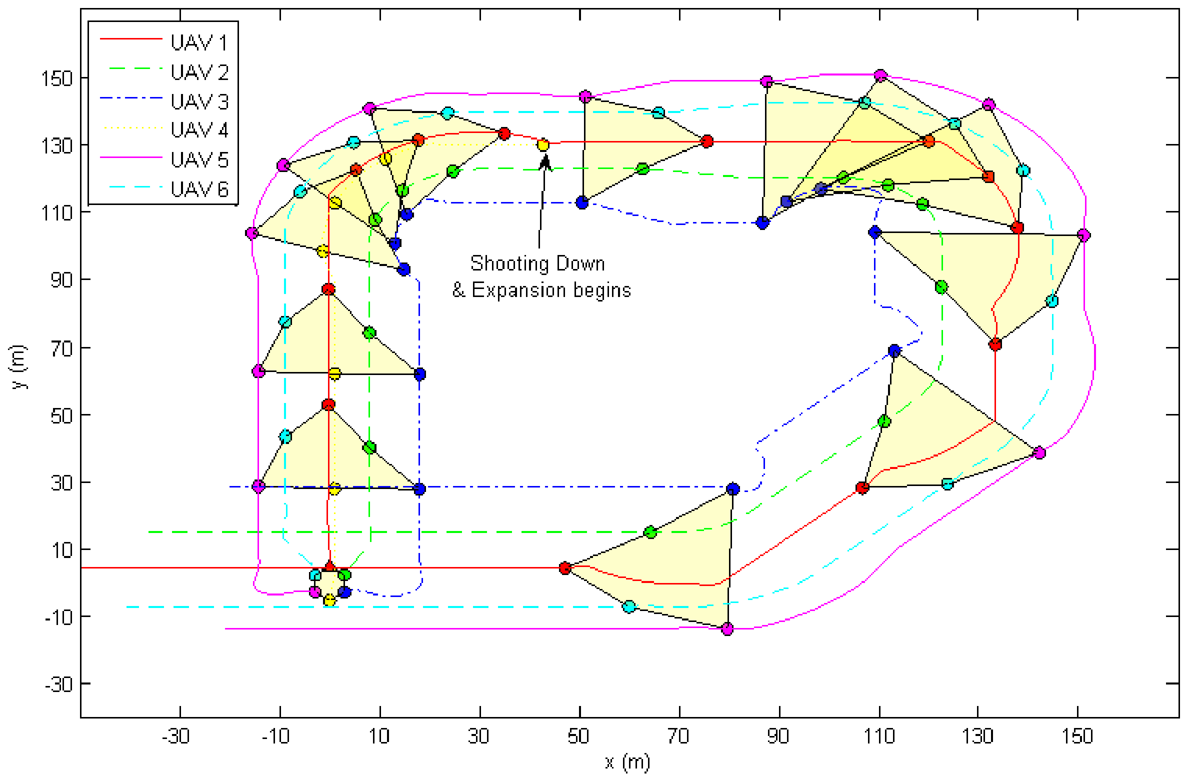

Fig. 4 Trajectories of each UAVs

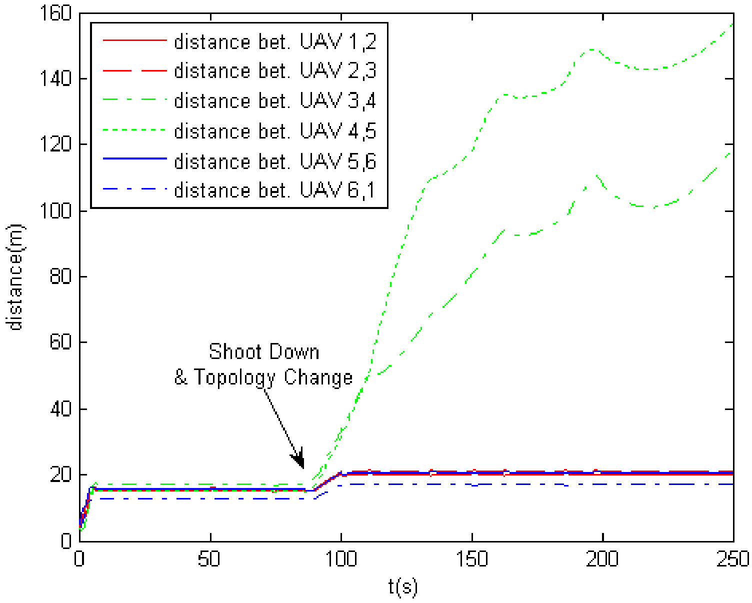

Fig. 5 Distance between each UAVs

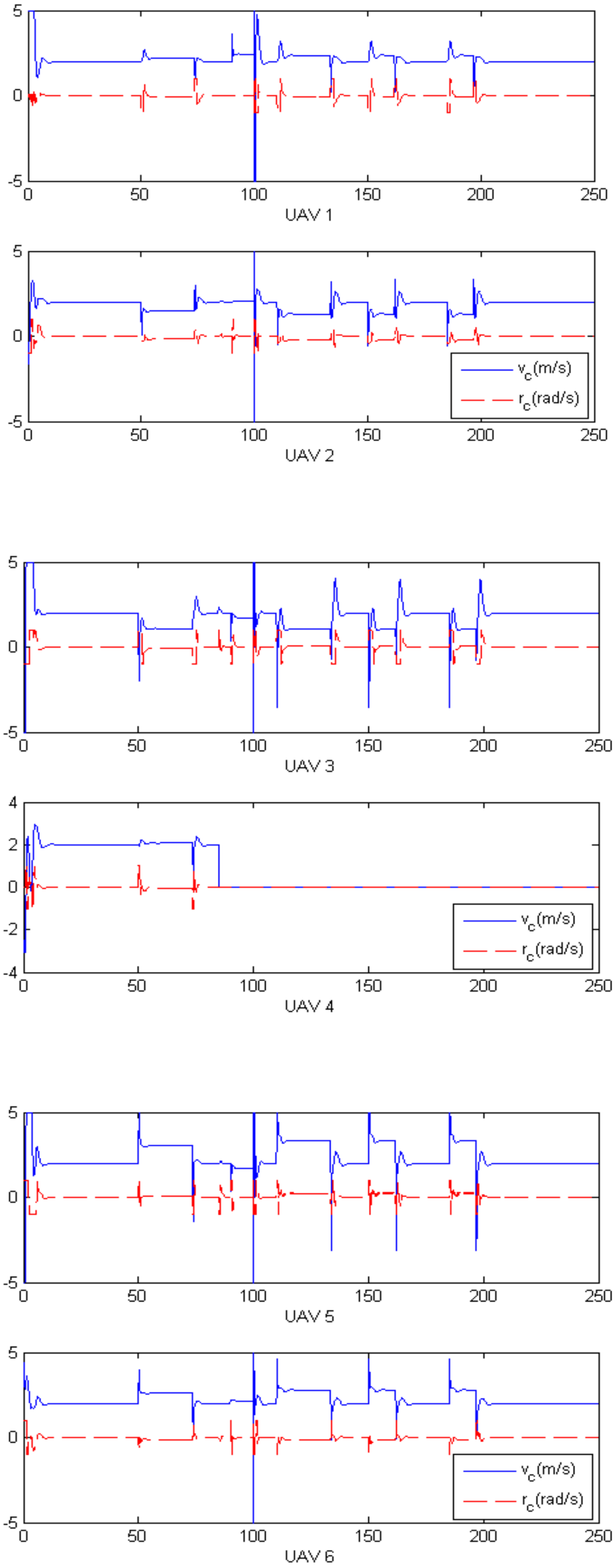

Fig.6 Control Input response of each UAVs

5. CONCLUSION

A consensus-based leaderless formation control strategy is proposed for multiple autonomous vehicles. In the proposed strategy, explicit leaders are not required in the team, and only local neighbor-to-neighbor information exchange is needed. Proposed strategy is applied to maintain a time-varying formation with a topology-switching for multiple UAVs. Stability analysis and numerical simulations are performed. The proposed strategy can also be applied to the satellite formation flight.

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REFERENCES

[1] J. R. Lawton, R. W. Beard, and B. Young, "A Decentralized Approach To Formation Maneuvers," *IEEE Transactions on Robotics and Automation*, Vol. 19, No. 6, 2003, pp. 933-941.

[2] R. O. Saber, R. M. Murray, "Agreement Problems in Networks with Directed Graphs and Switching Topology," *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, HI, December 2003.

[3] W. Ren, and Y. Q. Chen, "Leaderless Formation Control for Multiple Autonomous Vehicles," *AIAA Guidance, Navigation, and Control Conference*, Keystone, CO, August 2006.

[4] C. Godsil, and G. Royle, "*Algebraic Graph Theory, volume 207 of Graduate Texts in Mathematics*," Springer, 2001.

[5] W. Ren, and E. Atkins, "Second-order Consensus Protocols in Multiple Vehicle Systems with Local Interactions," *AIAA Guidance, Navigation, and Control Conference*, Keystone, CO, August 2006.

[6] A. Verma, and C. Wu, "Autonomous Command and Control System for UAV Formation", *AIAA Atmospheric Flight Mechanics Conference*, Austin, Texas, August 2003.