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Nonlinear Estimation for Spacecraft Attitude using Decentralized Unscented Information Filter

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Abstract: This paper presents a spacecraft attitude estimation algorithm using the unscented information filter. The objective of the proposed algorithm is to make the spacecraft attitude accurately estimate with low computational load. The main contribution of this study is to derive the unscented information filter for nonlinear spacecraft attitude dynamics and multiple sensor systems. For the multiple sensor estimation, the decentralized unscented information filter is used in this study. This filter provides accurate attitude estimation from the unscented filtering and less computational time from the information filtering. To verify the effectiveness of the proposed decentralized unscented information filter algorithm, numerical simulations are performed for a spacecraft system with three gyroscopes.

Keywords: Spacecraft, Attitude Estimation, Unscented Information Filter

1. INTRODUCTION

For a multiple sensor system, there are two different filter schemes for the measured sensor data process, centralized and decentralized filter scheme. In the centralized type, all measured sensor data are processed in the center site, and therefore the information loss can be minimized. However, the centralized filter scheme causes severe computational problem when the filter is overloaded, and therefore it may provide unreliable results. In the decentralized filter scheme, the local estimators of each sensor generate the global optimal or suboptimal state estimates according to the data fusion criterion. This filter scheme has some advantages; for example, the decentralized filter can treat lots of data because of the parallel structure. Moreover, the decentralized filter is robust to the sensor failure since each estimator deals with its own processing.

For the nonlinear spacecraft attitude estimation problems, there are several estimation algorithms including Kalman filter (KF), extended Kalman filter (EKF), unscented Kalman filter (UKF), particle filter, and information filter. EKF has been widely used to estimate the states of the spacecraft systems. In this scheme, the nonlinearities of the system are approximated by the first-order Taylor series expansion, and therefore it provides undesired estimates when the system has severe nonlinearities [1]. Recently, UKF has been studied because of the capability of capturing the posterior mean and covariance to the third order. UKF provides better attitude estimation results than EKF; however, UKF has some drawbacks including severe computational load. The information filter has been utilized in the multiple sensor fusion and the application of the decentralized filter scheme, because this filter has a number of benefits; the information filter achieves simpler estimation update and easier decoupling and decentralization than KF. Because there are no gain or innovation covariance matrices, the information filter is efficient in the multiple sensor systems [2]. The linear information filter is extended to a linearized estimation algorithm for nonlinear systems, which is the extended information filter (EIF). However, EIF has the truncation errors similar to EKF, and thus EIF cannot provide accurate state estimates for nonlinear spacecraft systems.

In this study, the unscented information filter (UIF) is derived for the spacecraft attitude estimation problem. The motivation of UIF algorithm comes from the unscented filtering for the nonlinear satellite system and information filtering for the multiple sensor systems. The proposed UIF provides more accurate estimate than the EIF and less computational time than the UKF. Also, the decentralized filter scheme is used for multiple sensor systems of the satellite attitude estimation. To verify the performance of the proposed algorithm, numerical simulations are performed for the spacecraft system.

This paper is organized as follows. In Section 2, the spacecraft attitude dynamics and the sensor modeling of gyroscope are described. The UIF algorithm and the decentralized UIF are derived in Section 3. Numerical simulation and analysis to verify the proposed algorithm are shown in Section 4. Finally, conclusions are presented in Section 5.

2. SPACECRAFT ATTITUDE DYNAMICS AND SENSOR MODELING

2.1 Spacecraft Attitude Kinematics and Dynamics

Quaternion is widely used to describe the attitude of a spacecraft, because it provides nonsingular attitude description and expresses arbitrary and large rotations of the satellite [1]. The quaternion is a four-dimensional vector defined as follows.

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \equiv \begin{bmatrix} \hat{q} \\ q_4 \end{bmatrix} \tag{1}$$

Note that the quaternion has the constraint, $q^T q = q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$.

The quaternion kinematic differential equation can be derived as follows.

$$\dot{q} = \frac{1}{2}\Xi(q)\omega = \frac{1}{2}\Omega(\omega)q \tag{2}$$

where ω is the three-dimensional rate vector, and

$$\Xi(q) = \begin{bmatrix} q_4 I_{3\times3} + \hat{q}^{\times} \\ -\hat{q}^T \end{bmatrix}, \quad \Omega(\omega) = \begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^T & 0 \end{bmatrix} \tag{3}$$

$$\hat{q}^{\times} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{4}$$

The composition of the quaternion is given by

$$q(t_1) \otimes q(t_2) = \left[\Xi(q(t_2)) \quad q(t_2)\right] q(t_1) \tag{5}$$

Also, the inverse quaternion is defined by

$$q^{-1} = \begin{bmatrix} -\hat{q} \\ q_4 \end{bmatrix} \tag{6}$$

Note that $q \otimes q^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$, which is the identity quaternion.

The attitude dynamics of a spacecraft can be represented from the Euler's momentum equation as follows [1].

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau \tag{7}$$

where J is a 3×3 moment of inertia matrix, and $\tau \in R^3$ is a control torque vector.

2.2 Sensor Modeling

In this study, a gyroscope is used as the attitude sensor of the spacecraft. The gyroscope is sensor that measures the angular rate of the spacecraft. The gyroscope system can be expressed mathematically [3] by modeling the measured angular velocity as the true angular velocity with an additive bias and Gaussian white-noise. The bias dynamics are considered to be driven by a Gaussian white-noise process. The gyroscope model can be represented as

$$\tilde{\omega} = \omega + \beta + \eta_v \tag{8}$$

$$\dot{\beta} = \eta_u \tag{9}$$

where $\tilde{\omega}$ is the measured angular velocity, ω is the true angular velocity, β is the drift, and η_v and η_u are independent zero-mean Gaussian white-noise processes with

$$E\left[\eta_v(t)\eta_v^T(\tau)\right] = \sigma_v^2 \delta(t-\tau) I_{3\times3} \tag{10}$$

$$E\left[\eta_u(t)\eta_u^T(\tau)\right] = \sigma_u^2 \delta(t-\tau) I_{3\times3} \tag{11}$$

where $E[\ ]$ denotes expectation, and $\delta(t-\tau)$ is the Dirac-delta function.

3. UNSCENTED INFORMATION FILTER

3.1 Unscented Information Filter

Consider the nonlinear system and measurement equation as follows.

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, t_k) + w_k, \quad & w_k &\sim N(0, Q_k) \\ z_k &= h(x_k, t_k) + v_k, & v_k &\sim N(0, R_k) \end{aligned} \tag{12}$$

where $x_k \in \Re^n$ is the state vector, $z_k \in \Re^m$ is the observation vector, $w_k \in \Re^n$ is the state noise vector, and $v_k \in \Re^m$ is the measurement noise vector. It is assumed that the noise vectors are uncorrelated white Gaussian process.

The information filter definition requires the introduction of new quantities, the Fisher information matrix and the information state vector as follows [2].

$$F_k \equiv Y_k = P_k^{-1} \tag{13}$$

$$\hat{y}_k = P_k^{-1}\hat{x}_k = Y_k \hat{x}_k \tag{14}$$

The UIF is derived based on the unscented transformation and information filtering approach. First, the time update of UIF algorithm is performed to propagate the state estimate and covariance from one measurement time to the next. To propagate from time step (*k-1*) to *k*, a set of sigma points is chosen using the current best guess of the mean and covariance as follows [4].

$$\begin{aligned} \hat{x}_{k-1}^i &= \hat{x}_{k-1}^+ + \tilde{x}^i \\ \tilde{x}^i &= \left(\sqrt{nP_{k-1}^+}\right)_i^T, \qquad & i &= 1,\dots,n \\ \tilde{x}^{n+i} &= -\left(\sqrt{nP_{k-1}^+}\right)_i^T, & i &= 1,\dots,n \end{aligned} \tag{15}$$

For the state propagation step, *a priori* state estimate $\hat{x}_k^-$ and error covariance P_k^- are estimated using the propagated sigma point vectors as

$$\hat{x}_k^- = \frac{1}{2n}\sum_{i=1}^{2n} \hat{x}_k^i \tag{16}$$

$$\hat{x}_k^i = f(\hat{x}_{k-1}^i, u_k, t_k) \tag{17}$$

$$P_k^- = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^i - \hat{x}_k^-\right)\left(\hat{x}_k^i - \hat{x}_k^-\right)^T + Q_{k-1} \tag{18}$$

The information prediction equations are derived by implementing Eqs. (16) ~ (18) as

$$\hat{y}_k^- = Y_k^- \hat{x}_k^- \tag{19}$$

$$Y_k^- = \left(P_k^-\right)^{-1} \tag{20}$$

Now, the measurement update is performed. The update equations for the information matrix and the information state vector are

$$Y_k^+ = Y_k^- + I_k \tag{21}$$

$$\hat{y}_k^+ = \hat{y}_k^- + i_k \tag{22}$$

where i_k is the information state contribution, and I_k is its associated information matrix as

$$i_k = \left(P_k^-\right)^{-1} P_{xz} R_k^{-1} z_k \tag{23}$$

$$I_k = \left(P_k^-\right)^{-1} P_{xz} R_k^{-1} \left(P_{xz}\right)^T \left(P_k^-\right)^{-T} \tag{24}$$

The predicted observation vector $\hat{z}_k$, its predicted measurement, and cross covariance between $\hat{x}_k^-$ and $\hat{z}_k$ are obtained as

$$\hat{z}_k = \frac{1}{2n}\sum_{i=1}^{2n} \hat{z}_k^i \tag{25}$$

$$\hat{z}_k^i = h(\hat{x}_k^i, t_k) \tag{26}$$

$$P_{xz} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^i - \hat{x}_k^-\right)\left(\hat{z}_k^i - \hat{z}_k\right)^T \tag{27}$$

$$P_{zz} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{z}_k^i - \hat{z}_k\right)\left(\hat{z}_k^i - \hat{z}_k\right)^T + R_k \tag{28}$$

Finally, the updated state estimate $\hat{x}_k^+$ and error covariance P_k^+ are given as follows.

$$\hat{x}_k^+ = \left(Y_k^+\right)^{-1} \hat{y}_k^+ \tag{29}$$

$$P_k^+ = \left(Y_k^+\right)^{-1} \tag{30}$$

3.2 Decentralized Unscented Information Filter

The UIF can be easily applied to the multiple sensor estimation problems [2]. For multiple sensor systems, the centralized and decentralized filter scheme based on UIF can be utilized. The centralized filter based on UIF provides accurate attitude estimates for spacecraft systems. However, this filter has a serious computational problem when the number of sensors is increased. The decentralized filter based on UIF is easy to implement, and it has less computational load than the centralized filter. The structures of the centralized and decentralized UIF are shown in Figs. 1 and 2. In this study, the decentralized UIF is used to deal with the multiple sensor estimation problems.

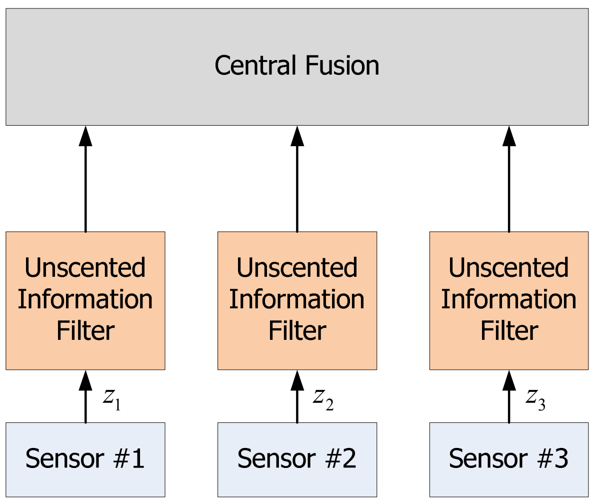

Fig. 1 The Structure of Centralized UIF (N=3)

Assume that the local filter estimates its predictions using Eqs. (19) and (20). Each local filter provides its local information state contribution $i_{j,k}$ and its associated information matrix $I_{j,k}$ using the sensor measurement at the j sensor as follows.

$$i_{j,k} = \left(P_{j,k}^-\right)^{-1} P_{j,xz} R_{j,k}^{-1} z_{j,k} \tag{31}$$

$$I_{j,k} = \left(P_{j,k}^-\right)^{-1} P_{j,xz} R_{j,k}^{-1} \left(P_{j,xz}\right)^T \left(P_{j,k}^-\right)^{-T} \tag{32}$$

where $P_{j,k}^-$, $P_{j,xz}$, and $z_{j,k}$ are *a priori* error covariance, cross covariance, and measurement of j local filter, respectively.

In this study, the network connected with N sensor nodes is considered, and it is assumed that the local sensors are communicated with each other as shown in Fig. 2. After communication, the estimation update based on decentralized UIF is performed for the multiple sensor estimation and data fusion. The update equations at each sensor node are given by combining with the local information contribution terms as follows.

$$\hat{y}_{i,k}^+ = \hat{y}_{i,k}^- + \sum_{j=1}^{N} i_{j,k} \tag{33}$$

$$Y_{i,k}^+ = Y_{i,k}^- + \sum_{j=1}^{N} I_{j,k} \tag{34}$$

Then, the global estimates, the updated state estimate, and error covariance in the fusion process can be calculated as follows.

$$\hat{y}_{k,fusion}^+ = \sum_{i=1}^{N} \hat{y}_{i,k}^+ \tag{35}$$

$$Y_{k,fusion}^+ = \sum_{i=1}^{N} \hat{Y}_{i,k}^+ \tag{36}$$

$$\hat{x}_k^+ = \left(Y_{k,fusion}^+\right)^{-1} \hat{y}_{k,fusion}^+ \tag{37}$$

$$P_k^+ = \left(Y_{k,fusion}^+\right)^{-1} \tag{38}$$

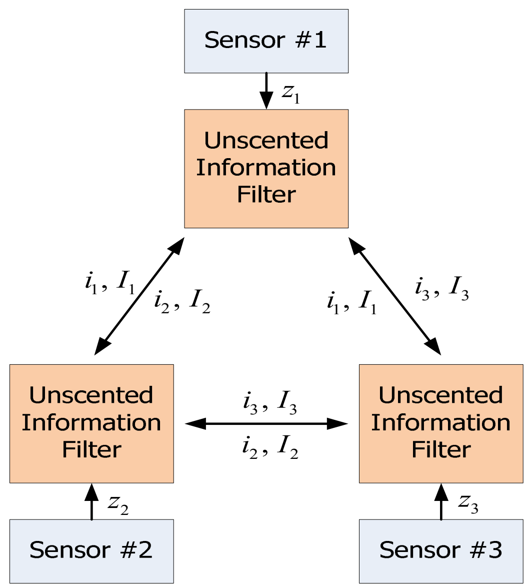

Fig. 2 The Structure of Decentralized UIF (N=3)

4. NUMERICAL SIMULATION

Numerical simulations are performed to verify the performance of the proposed decentralized UIF. For spacecraft attitude estimation, three gyroscopes are used as attitude sensors with the following initialization parameters.

$$\omega(t_0) = \begin{bmatrix} -0.01 & 0.06 & 0.001 \end{bmatrix}^T \text{ (deg/sec)} \tag{39}$$

The initial state estimate and covariance matrix are chosen as

$$\hat{x}(t_0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \tag{40}$$

$$P(t_0) = diag\left(\begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}\right) \tag{41}$$

and the process noise and measurement noise variance are given by

$$Q = diag\left(\begin{bmatrix} 10^{-5} & 10^{-5} & 10^{-5} \end{bmatrix}\right) \tag{42}$$

$$R = diag\left(\begin{bmatrix} 10^{-3} & 10^{-3} & 5\times10^{-3} \end{bmatrix}\right) \tag{43}$$

Simulation results are shown in Figs. 3~5. Figure 3 shows the time histories of the gyroscope measurement, Fig. 4 illustrates the estimated angular velocity history, and Fig. 5 shows the time histories of state estimation error, respectively. As shown in Figs. 4 and 5, the accurate state estimation is achieved by UIF.

Table 1 summarizes the performance of the proposed decentralized UIF algorithm compared with the decentralized UKF [4] in terms of the computational load and estimation accuracy. As shown in Table 1, the proposed decentralized UIF results in 36% reduction in computational time compared with the decentralized UKF. It is because the decentralized filter scheme and information filtering help to reduce the computational load for the multiple sensor estimation. Moreover, the decentralized UIF provides better performance than the decentralized UKF in the terms of the attitude accuracy.

Table 2 shows the performance of the decentralized UIF and UKF for the system with ten attitude sensors (N=10). Note from Table 2 that the computational time of the decentralized UIF is much less than that of the decentralized UKF. Even though the decentralized UKF has serious computational load by increasing the number of sensors, the decentralized UIF can handle much more data because of the information filtering. From the numerical simulation results, it can be seen that the decentralized UIF improves the estimation accuracy and computational efficiency for the multiple sensor estimation problems.

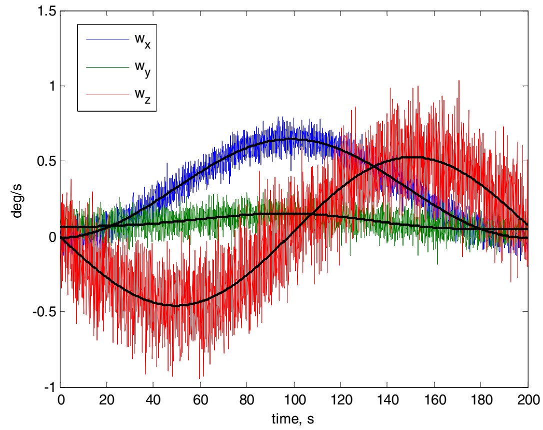

Fig. 3 Angular Velocity Measurement

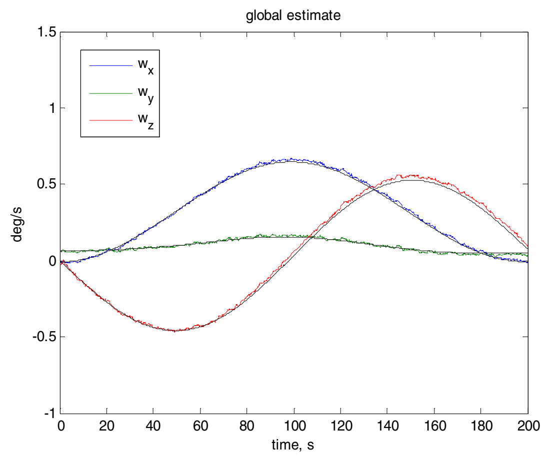

Fig. 4 Angular Velocity Estimation

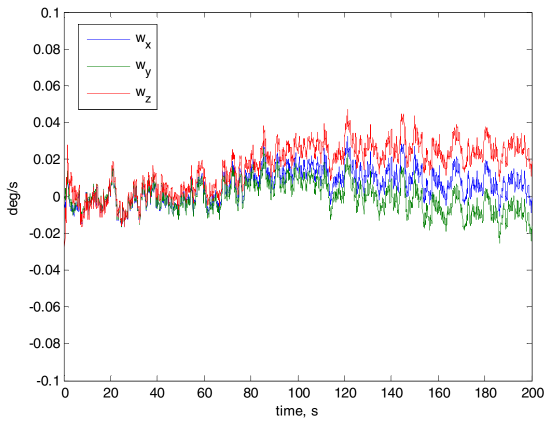

Fig. 5 Estimation Error

Table 1 Computational Time and Error Norm (N=3)

	Computational Time (s)	Error Norm
DUKF	29.301614	0.0404
DUIF	18.876301	0.0254

Table 2 Computational Time and Error Norm (N=10)

	Computational Time (s)	Error Norm
DUKF	97.808695	0.1146
DUIF	70.688573	0.0359

5. CONCLUSION

Nonlinear estimation algorithm for the spacecraft system is proposed. To estimate the spacecraft attitude, UIF algorithm is presented using the unscented transformation for the nonlinear system and the information filtering for the multiple sensor system. The decentralized UIF is also utilized for multiple sensor estimation problems. The decentralized filter scheme provides simple implementation and less computational load compared with the centralized filter scheme. Numerical simulation results show that the proposed decentralized UIF algorithm provides the accurate attitude estimation and less computational time compared with the centralized UIF and the decentralized UKF. The proposed algorithm can be applied for the ground mobile robots and aerial robot systems as well as the satellite systems.

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